Ch.1 - Sets

WORKSHEET-I

1. Describe each of the following sets in Roster from

(i) $\{x : x$ is a positive integer and a divisor of 9$\}$ [{1, 3, 9}]

(ii) $\{x : x \in Z \ and \ |x| \leq 2\}$ [$x = 0, \pm 1, \pm 2$]

(iii) {x: x is a letter of the word 'PROPORTION'} [{P, R, O, T, I, N}]

(iv) $\left\{x : x = \frac{n}{n^2+1} \ and \ 1 \leq n \leq 3, \ where \ n \in N\right\}$ [$\left\{\frac{1}{2}, \frac{2}{5}, \frac{3}{10}\right\}$]

2. Write the set $\left\{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{7}{8}, \frac{8}{9}, \frac{9}{10}\right\}$ in the set-builder from.

[$X = \left\{1, \frac{1}{4}, \frac{1}{9}, \frac{1}{16}, \frac{1}{25}, ...\right\}$ in the set builder from.]

3. Describe the following sets in Roster form:

(i) $\{x : x$ is a letter before e in the English alphabet$\}$. [$\{a, b, c, d\}$]

(ii) $\{x \in N : x^2 < 25\}$ [{1, 2, 3, 4}]

(iii) $\{ \ x \in N : x$ is a prime number, $10 < x < 20 \}$ [{11, 13, 17, 19},]

(iv) $\{x \in N : x = 2n, n \in N\}$ [{2, 4, 6, 8, …….}]

(v) $\{x \in N : x > x\}$ [ϕ]

(vi) $\{x : x$ is a prime number which is a divisor of 60$\}$. [{2, 3, 5}]

(vii) $\{x : x$ is a two digit number such that the sum of its digits is 8$\}$

[{17, 26, 35, 44, 53, 62, 71, 80}]

(viii) The set of all letters in the word 'Trigonometry'. [{17, 26, 35, 44, 53, 62, 71, 80}]

(ix) The set of all letters in the word 'Better'.

[{T, R, I, G, O, N, M, E, Y}(ix){B, E, T, R}]

4. Describe the following sets in set-builder form:

(i) a = { 1, 2, 3, 4, 5, 6} [$\{x; x \in N, x < 7\}$]

(ii) B = {1, 1/2, 1/3, 1/4, 1/4,…..} [$[x : x = 1/n, x \in n]$]

(iii) C = {0, 3, 6, 9, 12,……} [$[x : x = 3n, n \in Z^+]$]

(iv) D = {10, 11, 12, 13, 14, 15} [$\{x : x \in N, 9 < x < 16\}$]

(v) E = {0} [$\{x : x = 0\}$]

(vi) {1, 4, 9, 16,……., 100} [$\{x^2 : x \in N, 1 \leq x \leq 10\}$]

(viii) {2, 4, 6, 8,……..} [$\{x : x = 2n, n \in N\}$]

(viii) {5, 25, 125, 625} [$\{5^n : n \in N, 1 \le N \le 4\}$]

5. List all the elements of the following sets:

(i) $A = \{x : x^2 \le 10, x \in Z\}$ [A = $\{0, \pm1, \pm2, \pm3\}$]

(ii) $B = \left\{x : x = \dfrac{1}{2n-1}, 1 \le n \le 5\right\}$ [$B = \left\{1, \dfrac{1}{3}, \dfrac{1}{5}, \dfrac{1}{7}, \dfrac{1}{9}\right\}$]

(iii) $\left\{C = x : x \text{ is an integer}, \dfrac{1}{2} < x < \dfrac{9}{2}\right\}$ [C = {0, 1, 2, 3, 4}]

(iv) D = {*x* : *x* is a vowel in the word "EQUATION"} [X = { A, E, I, O , U}]

(v) E = {x : x is a month of a year not having 31 days}

[E = { Feb., April, June, Sept., November}]

(vi) F { x : x is a letter of the word "MISSISSIPPI"} [F = {M, I, S, P}]

6. Write the set $\left\{\dfrac{1}{2}, \dfrac{2}{5}, \dfrac{3}{10}, \dfrac{4}{17}, \dfrac{5}{26}, \dfrac{6}{37}, \dfrac{7}{50}\right\}$ in the set-builder form. [$\left\{\dfrac{n}{n^2+1} : n \in N, n \le 7\right\}$]

7. Which of the following sets are finite and which are infinite?

(i) Set of concentric circles in a plane. [Infinite]

(ii) Set of letters of the English Alphabets [Finite]

(iii) $\{x \in N : x > 5\}$ [Infinite]

(iv) $\{x \in N : x < 200\}$ [Finite]

(v) $\{x \in Z : x < 5\}$ [Infinite]

(vi) $\{x \in R : 0 < x < 1\}$ [Infinite]

8. Which of the following sets are equal?

(i) A = {1, 2, 3}

(ii) $B = \{x \in R : x^2 - 2x + 1 = 0\}$

(iii) C = {1, 2, 2, 3}

(iv) $D = \{x \in R : x^3 - 6x^2 + 11x - 6 = 0\}$ [A = C = D]

9. Two finite sets have m and n elements. The total number of subsets of the first set is 56 more than the total number of subsets of the second set. Find the values of m and n. [$X \subset Y$]

10. Write the following subsets of R as intervals:

(i) $\{x : x \in R, -4 < x \le 6\}$ [$\{x : x \in R, -4 < x \le 6\} = (-4, 6]$ Length = 6 –(–4) = 10]

(ii) $\{x : x \in R, -12 < x < -10\}$

[$\{x : x \in R, -12 < x < -10\}$ = (–12, 10). Length = –10 – (–12) = 2]

(iii) $\{x : x \in R, 0 \le x < 7\}$ [$\{x : x \in R, 0 \le x < 7\} = [0,7)$. Length = 7 – 0 = 7]

(iv) $\{x : x \in R, 3 \le x \le 4\}$ [$\{x : x \in R, 3 \le x \le 4\} = [3,4]$. Length = 4 – 3 = 1]

Also, find the length of each interval.

11. Write the following intervals in the set-builder form:

(i) (–7. 0) [$(-7,0) = \{x : x \in R \text{ and } -7 < x < 0\}$]

(ii) [6, 12] [$[6,12] = \{x : x \in R \text{ and } 6 \le x \le 12\}$]

(iii) (6, 12] [$(6,12] = \{x : x \in R \text{ and } 6 < x \le 12\}$]

(iv) [–20, 3) [$[-20,3) = \{x : x \in R \text{ and } -20 \le x < 3\}$]

12. Let A = {a, b, {c, d}, e}. Which of the following statements are false and why?

(i) $\{c.d\} \subset A$ [F]

(ii) $\{c, d\} \in A$ [T]

(iii) $\{\{c, d\}\} \subset A$ [T]

(iv) $a \in A$ [T]

(v) $a \subset A$ [F]

(vi) $\{a, b, e\} \subset A$ [T]

(vii) $\{a, b, e\} \in A$ [F]

(viii) $\{a, b, c\} \subset A$ [F]

(ix) $\phi \in A$ [T]

(x) $\{\phi\} \subset A$ [F]

13. Let $A = \{\{1, 2, 3\}, \{4, 5\}, \{6, 7.8\}\}$. Determine which of the following is true or false.

(i) $1 \in A$ [F]

(ii) $\{1, 2, 3\} \subset A$ [F]

(iii) $\{6, 7, 8\} \in A$ [T]

(iv) $\{\{4, 5\}\} \subset A$ [T]

(v) $\phi \in A$ [F]

(vi) $\phi \subset A$ [T]

14. Let $A = \{\phi, \{\phi\}, 1, \{1, \phi, 2\}$. Which of the following are true?

(i) $\phi \in A$ [T]

(ii) $\{\phi\} \in A$ [T]

(iii) $\{1\} \in A$ [F]

(iv) $\{2, \phi\} \subset A$ [T]

(v) $2 \subset A$ [F]

(vi) $\{2, \{1\}\} \not\subset A$ [T]

(vii) $\{\{2\}, \{1\}\} \not\subset A$ [T]

(viii) $\{\phi, \{\phi\}, \{1, \phi\}\} \subset A$ [T]

(ix) $\{\{\phi\}\} \subset A$ [T]

15. Write down all possible subsets of each of the following sets:

(i) {a} [$\phi, \{a\}$]

(ii) {0, 1} [$\phi, \{0\}, \{1\}, \{0,1\}$]

(iii) {a, b, c} [$\phi, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{a,b,c\}$]

(iv) {1, {1}} [$\phi, \{1\}, \{\{1\}\}, \{1, \{1\}\}$]

16. If A and B are two sets such that $A \subset B$, then find:

(i) $A \cap B$ (ii) $A \cup B$ [(i)A (ii)B]

17. Let $A = \{x : x \in N\}, B\{x : x = 2n, n \in N\}, C = \{x : x = 2n - 1, n \in N\}$

and, $D = \{x : x \ is \ a \ \ prime \ natural \ \ number\}$. Find

(i) $A \cap B$ (ii) $A \cap C$

(iii) $A \cap D$ (iv) $B \cap C$

(v) $B \cap D$ (vi) $C \cap D$

[(i)B (ii)C (iii)D (iv) ϕ (v) $(A')'$ (vi) $(B - C)'$]

18. Let U = $\{1, 2, 3, 4, 56, 7, 8, 9\}$, $A = \{1, 2, 3, 4\}$, $B = \{2, 4, 8\}$ and $C = \{3, 4, 5, 6\}$

(i) A' [{5, 6, 7, 8, 9}]

(ii) B' [{1, 3, 5, 7, 9}]

(iii) $(A \cap C)'$ [{1, 2, 5, 6, 7, 8, 9}]

(iv) $(A \cap B)'$ [{5, 7, 9}]

(v) $(A')'$ [A]

(vi) $(B - C)'$ [{1, 3, 4, 5, 6, 7, 9}]

19. Let A, B and C be the sets such that $A \cup B = A \cup C$ and $A \cap B = A \cap C$. Show that B = C.

20. Using properties of sets, show that for any tow sets A and B, $(A \cup B) \cap (A \cup B') = A$.

[$(A \cup B) \cap (A \cup B') = ((A \cup B) \cap A) \cup ((A \cup B) \cup B')$

$= (A \cup ((A \cup B) \cap B') = A \cup (A \cap B') \cup (B \cap B') \ A \cup (A \cap B') = A$]

21. Show that for any sets A and B,

(i) $A = (A \cap B) \cap (A - B)$

(ii) $A \cup (B - A) = A \cup B$

22. In a group of 50 people, 35 speak Hindi, 25 speak both English and Hindi and all the people speak at least one of the two languages. How many people speak only English and not Hindi? How many people speak English? [40]

23. There are 200 individuals with a skin disorder, 120 has been exposed to chemical C_1, 50 to chemical C_2 and 30 to both the chemicals C_1 and C_2. Find the number of individuals exposed to (i) Chemical C_1 or chemical C_2 (ii) Chemical C_1 but not chemical C_2 (iii) Chemical C_2 but not chemical C_1.

[(i)140 (ii)90 (iii)20]

24. If A and B be two sets containing 3 and 6 elements respectively, when can be the minimum number of elements in $A \cup B$? Find Also, the maximum number of elements in $A \cup B$

[Maximum number of elements in $A \cup B$ = 9.

Minimum number of elements in $A \cup B$ = 6]

25. There are 40 students in a Chemistry class and 650 students in a Physics class. Find the number of students which are either in Physics class or Chemistry class in the following cases:

(i) The two classes meet at the same hour.

(ii) the two classes meet at different hours and 20 students are enrolled in both the subjects.

[(i)100 (ii)80]

26. In a survey of 700 students in a college, 180 were listed as drinking Limca, 275 as drinking Miranda and 95 were listed as both drinking Limca as well as Miranda. Find how many students were drinking neither Limca nor Mirnda. [340]

27. A survey shows that 63% of the Americans like cheese whereas 76% like apples. If x% of the Americans like both cheese and apples, find the value of x. [$39 \le x \le 63$]

28. In a class of 35 students, 17 have taken Mathematics, 10 have taken Mathematics but not Economics. Find the number of students who have taken both Mathematics and Economics and the number of students who have taken Economics but not Mathematics, if it is given that each student has taken either Mathematics or Economics or both. [18]

29. In a town of 10,000 families it was found that 40% families buy newspaper A, 20% families buy newspaper B and 10% families buy newspaper C. 5% families buy A and B , 3% buy B and C and 4% buy A and C. If 2% families buy all the three news papers, find the number of families which buy (i) A only (ii) B only (iii) none of A, B and C. [(i)3300 (ii)1400(iii)4000]

30. A college awarded 38 medals in Football, 15 in Basketball and 20 to Cricket. If these medals went to a total of 58 men and only three men got medals in all the three sports, how many received metals in exactly two of the three sports? [9]

31. In a survey of 25 students, it was found that 15 had taken Mathematics, 12 had taken Physics and 11 had taken Chemistry, 5 had taken Mathematics and Chemistry, 9 had taken Mathematics and Physics, 4 had taken Physics and Chemistry and 3 had taken all the three subjects. Find the number of students that had

(i) Only Chemistry

(ii) Only Mathematics

(iii) Only Physics

(iv) Physics and Chemistry but not Mathematics

(v) Mathematics and Physics but not Chemistry

(vi) Only one of the subjects

(vii) At least one of the three subjects

(viii) None of the subjects. [(i)5 (ii)4 (iii)2 (iv)1 (v)6 (vi)11 (vii)23 (viii)2]

32. Let A and B be two sets such that : $n(A)=20$, $n(A\cup N)=42$ and $n(A\cap B)=4$. Find

(i) n(B)

(ii) n(A – B)

(iii) n (B – A) [(i) 26 (ii) 16 (iii) 22]

33. In a survey of 60 people, it was found that 25 people read newspaper H, 26 read newspapers T, 26 read newspaper I, 9 read both H and I, 11 read both H and T, 8 read both T and I, 3 read all three newspapers. Find:

(i) The numbers of people who reads at least one of the newspapers.

(ii) The number of people who read exactly one newspaper. [(i)52 (ii) 30]

34. Of the members of three athletic teams in a certain school, 21 are in the basketball team, 26 in hockey team and 29 in the football team. 14 play hockey and basketball, 15 play hockey and football, 12 play football and basketball and 8 play all the three games. How many members are there in all? [43]

35. In a survey of 10 persons it was found that 28 read magazine A, 30 read magazine B, 42 read magazine C, 8 read magazines A and B, 10 read magazines A and C, 5 read magazines B and C and 3 read all the three magazines. Find

(i) How many read none of three magazine?

(ii) How many read magazine C only? [(i) 20 (ii) 30]

WORKSHEET-I

1. Find x and y, if (x + 3, 5) = (6, 2x + y) [–1]

2. Let A = {1, 2, 3} and $B = \{x : x \in N, x\, is\ prime\ less\ than\ 5\}$. Find A ×B and B × A.

   [A × B = {1, 2, 3} × {2, 3} = {(1, 2), (1, 3) , (2, 2) (2, 3), (3, 2), (3, 3)}

   B × A = { 2, 3} × { 1, 2, 3} = {(2, 1), (2, 2) (2, 3), (3, 1), (3, 2), (3, 3)}]

3. If $A \times B = \{(a,1),(b,3),(a,3)(b,1),(a,2),(b,2)\}$, find A and B.

   [A = {a, b} and B = {1, 2, 3}]

4. Let A and B be two sets such that A × B consist of 6 elements. If three elements of A × B are : (1, 4), (2, 6), (3, 6). Find A × B and B × A.

   [A × B = {1, 2, 3} × {4, 6} = {(1, 4), (1, 6), (2, 4), (2, 6), (3, 4), (3, 6)}

   B × A = {4, 6} × {1, 2, 3} = {(4, 1), (4, 2), (4, 3), (6, 1), (6, 2), (6, 3)}]

5. The Cartesian produce A × A has 9 elements among which are found (–1, 0) and (0, 1). Find the set A and the remaining elements of A × A. [$A = \{-1, 0, 1\}$]

6. Let A and B be two sets such that n (A) = 5 and n (B) = 2. If a, b, c, d, e are distinct and (a, 2), (b, 3) , (c, 2), (d, 3), (e, 2) are in A × B, find A and B. [A = {a, b, c, d, e}, B = {2, 3}]

7. Let $A = \{-1, 3, 4\}$ and $B = \{2, 3\}$. Represent the following products graphically i.e. by lattices:

   (i) A × B (ii) B × A (iii) A × A

8. If the ordered pairs (x, –1) and (5, y) belong to the set {(a, b): b = 2a –3}, find the values of x and y.

   [X = 1, y = 7]

9. If $a \in \{-1, 2, 3, 4, 5\}$ and $b \in \{0, 3, 6\}$, write the set of all ordered pairs (a, b) such that a + b = 5.

   [{(–1, 6), (2, 3), (5, 0)}]

10. If $a \in \{2, 4, 6, 9\}$ and $b \in \{4, 6, 18, 27\}$, then form the set of all ordered pairs (a, b) such that a divides b and a < b. [{(2, 4), (2, 6), (2, 18), (6, 18), (9, 18), (9, 27)}]

11. If $A = \{1, 2, 3\}$ and $B = \{2, 4\}$, what are A × B, B × A, A × A, B × B, and $(A \times B) \cap (B \times A)$?

    [{(2, 2)}]

12. If A and B are two sets having 3 elements in common. $If n(A) = 5$, $n(B) = 4$, find $n(A \times B)$ and $n[(A \times B) \cap (B \times A)]$ [$n(A \times B) = 20, n[(A \times B) \cap (B \times A)] = 9$]

13. If $A = \{-1, 1\}$, find A × A × A.

    [$A \times A \times A = \{(-1,-1,-1,),(-1,-1,1),(-1,1,-1), (-1,1,1),(1,-1,-1),(1,-1,1), (1,1,-1),(1,1,1)\}$]

14. Let A = {1, 2}, B = {1, 2, 3,4 }, C = {5, 6} and D = {5, 6, 7, 8}. Verify that:

    (i) $A \times C \subset B \times D$

    (ii) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

    [$R = \{(1,1),(1,2),(1,3),(1,4),(2,2)(2,4),(3,3),(4,4)\}$]

15. If $A = \{1, 3, 5, 7\}, B = \{2, 4, 6, 8, 10\}$ and let $R = \{(1,8),(3,6),(5,2),(1,4)$ be a relation from A to B. Then,

    Domain (R) = {1, 3, 5} and Range (R) = {8, 6, 2, 4}

16. Let $A = \{1, 3, 5, 7\}$ and $B = \{2, 4, 6, 8\}$ be two sets and let R be a relation from A to B defined by the phrase $"(x, y) \in R \Leftrightarrow x > y"$. Under this relation R, we obtain 3R2, 5R2, 5R4, 7R2, 7R4 and 7R6.

17. Let A be the set of first then natural numbers and let R be a relation on A defined by $(x, y) \in R \Leftrightarrow x + 2y = 10$ i.e. $R = \{(x, y) : x \in A, y \in A \text{ and } x + 2y = 10\}$. Express R and R^{-1} assets of ordered pairs. Also, determine (i) domains of R and R^{-1} (ii) Ranges of R and R^{-1}. [Dom (R^{-1}).]

18. A relation R is defined on the set Z of integers as: $(x, y) \in R \Leftrightarrow x^2 + y^2 = 25$ [Dom(R^{-1}).]

19. Let R be the relation on the set N of natural numbers defined by $R = \{(a, b) : a + 3b = 12, a \in N\}$

    Find

    (i) R

    (ii) Domain of R

    (iii) Range of R

    [(i) R = {(9, 1), (6, 2), (3, 3) (ii) Domain of R = {9, 6, 3}(iii) Range of R = {1, 2, 3}]

20. Let $A = \{1, 2, 3, 4, 5, 6\}$. Define a relation R on set A by R = $\{(x, y) : y = x + 1\}$

    (i) Depict this relation using an arrow diagram [$R = \{(1, 2,), (2, 3), (3, 4), (4, 5), (5, 6)\}$]

    (ii) Write down the domain, co-domain and range of R.

    [Domain (R) = {1,2,3,4,5} Range (R) = {2,3,4,5,6}]

21. Determine the domain and range of the relation R defined by

    (i) $R = \{(x, x + 5) : x \in \{0, 1, 2, 3, 4, 5\}\}$

    [Domain $R = \{0, 1, 2, 3, 4, 5\}$, Range $R = \{5, 6, 7, 8, 9, 10\}$

    (ii) $R = \{(x, x^3) : x$ is a prime number less than 10}

    [Domain $R = \{2, 3, 5, 7\}$, Range $R = \{8, 27, 125, 343\}$]

22. Let $A = \{a, b\}$. List all relations on A and find their number [16]

23. Let $A = \{x, y, z\}$ and $B = \{a, b\}$. Find the total number of relations from A into B. [64]

24. Let $A = \{1, 2, 3, ..., 14\}$. Define a relation on a set A by

    $R = \{(x, y) : 2x - y = 0$, where $x, y \in A\}$

    Depict this relationship using an arrow diagram. Write down its domain, co-domain and range.

    [Domain (R) = {1, 2, 3, 4}, Co-domain (R) = A, Range (R) = {3, 6, 9, 12}]

25. Define a relation R on the set N of natural numbers by

    $R = \{(x, y) : y = x + 5$, x is a natural number less than $4, x, y \in N\}$

    Depict this relationship using

    (i) roster form [$R = \{1, 6), (2, 7), (3, 8)$]

    (ii) an arrow diagram. Write down the domain and range or R

    [Domain $(R) = [1, 2, 3]$, Range $(R) = \{6, 7, 8\}$]

26. Let $A = \{1, 2, 3, 4, 5, 6\}$. Let R be a relation on A defined by

    $R = \{(a, b) : a, b \in A, b$ is exactly divisible by a}

    (i) Write R in roster form

[R = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 2), (2, 4), (2, 6), (3, 3), (3, 6), (4, 4), (5, 5), (6, 6)]

(ii) Find the domain of R [Domain (R) = {1, 2, 3, 4, 5, 6}]

(iii) Find the range of R. [Range (R) = {1, 2, 3, 4, 5, 6}]]

27. Let R be the relation on Z defined by $R=\{(a,b): a,b\in Z,\ a-b$ is an integer}. Find the domain and range of R. [(i) Domain (R) = Z, Range (R) =Z.]

28. Find the domain for which the function $f(x)=2x^2-1$ and $g(x)=1-3x$ are equal. [$\{-2, 1/2\}$]

29. Is $g=\{(1,1),(2,3),(3,5),(4,7)\}$ a function? If this is described by the formula, $g(x)=\alpha x+\beta$, then what values should be assigned to α and β? [$\alpha = 2$, $\beta=-1$]

30. Let $f=\{(1,1),(2,3),(0,-1),(-1,-3)\}$ be a function described by the formula $f(x)=ax+b$ for some integers a, b. Determine a, b. [a = 2 and b = –1.]

31. If $f: R\to R$ be defined as follows: $f(x)=\begin{cases}1, & if\ x\in Q\\ -1, & if\ x\notin Q\end{cases}$

Find (i) $f(1/2), f(\pi), f(\sqrt{2})$ (ii) Range of *f*.

(iii) pre-image of 1 and –1

[(i) –1 (ii) $f=\{1,-1\}$ (iii) $f^{-1}(-1)=R-Q$ =Set of irrational numbers.]

32. If a function $f: R\to R$ be defined by

$$f(x)=\begin{cases}3x-2 & , \quad x<0\\ 1 & , \quad x=0\\ 4x+1 & , \quad x>0\end{cases}$$

Find: $f(1), f(-1), f(0), f(2)$

[$f(1)=5,\ f(-1)=-5, f(0)=1, f(2)=9$]

33. If $f(x)=3x^4-5x^2+9$, find $f(x-1)$ [$3x^4-12x^3+13x^2-2x++7$]

34. If $f(x)=x+\frac{1}{x}$, prove that $[f(x)]^3=f(x^3)+3f\left(\frac{1}{x}\right)$

35. If $f(x)=\frac{1}{2x+1}$, $x\neq -\frac{1}{2}$, then show that $f(f(x))=\frac{2x+1}{2x+3}$, provide that $x\neq -\frac{3}{2}$.

36. If *f* is a real function defined by $f(x)=\frac{x-1}{x+1}$, then prove that: $f(2x)=\frac{3f(x)+1}{f(x)+3}$

37. If $f(x)=(a-x^n)^{1/n}$, a > 0 and $n\in N$, then prove that $f(f(x))=x$ for all *x*.

38. If $f(x)=x^3-\frac{1}{x^3}$, show that $f(x)+f\left(\frac{1}{x}\right)=0$.

39. If for non-zero $x, af(x)+b\ f\left(\frac{1}{x}\right)=\frac{1}{x}-5$, where $a\neq b$, then find *f*(*x*). [$\frac{1}{a^2-b^2}\left\{\frac{a}{x}-bx\right\}-\frac{5}{a+b}$]

40. Find the domain of each of the following real valued functions:

(i) $f(x)=\dfrac{1}{x+2}$ [Domain $(f)=R-\{-2\}$]

(ii) $f(x)=\dfrac{x-1}{x-3}$ [Domain $(f)=R-\{3\}$]

(iii) $f(x)=\dfrac{2x-3}{x^2-3x+2}$ [Domain $(f)=R-\{1,2\}$]

(iv) $f(x)=\dfrac{x^2+3x+5}{x^2-5x+4}$ [Domain (F) =[–2, 2]]

41. Find the domain of each of the following functions:

(i) $f(x)=\sqrt{x-2}$ [Domain $(f)=[2,\infty)$]

(ii) $f(x)=\dfrac{1}{\sqrt{1-x}}$ [Domain $(f)=(-\infty,1)$]

(iii) $f(x)=\sqrt{4-x^2}$ [Domain $(f)=[-2,2]$]

42. Find the domain of the function $f(x)$ defined by $f(x)=\sqrt{4-x}+\dfrac{1}{\sqrt{x^2-1}}$

[Domain $(f)=(-\infty,-1)\cup(1,4]$]

43. Find the domain and range of the function $f(x)$ given by $f(x)=\dfrac{x-2}{3-x}$

[Domain $(f)=R-\{3\}$, Range $(f)=R-\{-1\}$.]

44. Find the range of each of the following functions:

(i) $f(x)=\dfrac{1}{\sqrt{x-5}}$ [Domain $(f)=(5,\infty)$, Range $(f)=(0,\infty)$]

(ii) $f(x)=\sqrt{16-x^2}$ [Domain $(f)=[-4,4]$, Range $(f)=[0,4]$]

(iii) $f(x)=\dfrac{x}{1+x^2}$ [Domain $(f)=R$, Range $(f)=[-1/2,1/2]$]

(iv) $f(x)=\dfrac{3}{2-x^2}$ [Domain $(f)=R-\{-\sqrt{2},\sqrt{2}\}$, Range $(f)=(-\infty,0)\cup[3/2,\infty)$]

45. Find the domain and range of the function $f(x)=\dfrac{x^2-9}{x-3}$

[Domain $(f)=R-\{3\}$, Range $(f)=R-\{6\}$]

46. Find the Domain and Range of the function $f(x)=\dfrac{1}{2-\sin 3x}$

[Domain $(f)=R$, Range $(f)=[1/3,1]$]

47. Find the domain and range of each of the following real valued functions:

(i) $f(x)=\dfrac{ax+b}{bx-a}$ [Domain $R-\left\{\dfrac{a}{b}\right\}$ Range $R-\left\{\dfrac{a}{b}\right\}$]

(ii) $f(x)=\dfrac{ax-b}{cx-d}$ [Domain $R-\left\{\dfrac{d}{c}\right\}$ Range $R-\left\{\dfrac{a}{c}\right\}$]

(iii) $f(x)=\sqrt{x-1}$ [Domain $[1,\infty)$ Range $[0,8)$]

(iv) $f(x)=\sqrt{x-3}$ [Domain $[3,\infty)$ Range $[0,8)$]

(v) $f(x)\frac{x-2}{2-x}$ [Domain $R-[2]$ Range $\{-1\}$]

(vi) $f(x)=|x-1|$ [Domain R Range $[0,8)$]

(vii) $f(x)=-|x|$ [Domain R Range $(-\infty,0)$]

(viii) $f(x)=\sqrt{9-x^2}$ [Domain [–3, 3}Range[0, 3]]

48. Let *f* and *g* be two real functions defined by $f(x)=\frac{1}{x+4}$ and $g(x)=(x+4)^3$.

Find the following:

(i) $f+g$ [$\frac{(x+4)^4}{x+4}$]

(ii) $f-g$ [$\frac{1-(x+4)^4}{x+4}$]

(iii) fg [$(x+4)^2$]

(iv) $\frac{f}{g}$ [$R-\{-4\}$]

(v) 2f [$x\in R-\{-4\}$]

(vi) $\frac{1}{f}$ [$(x+4)$]

(vii) $\frac{1}{8}$ [$\frac{1}{(x+4)^3}$]

49. Find the domain of each of the following functions given by

(i) $f(x)=\frac{1}{\sqrt{x-|x|}}$ [Domain $(f)=\phi$]

(ii) $f(x)=\frac{1}{\sqrt{x+|x|}}$ [Domain $(f)=(0,\infty)$]

(iii) $f(x)+\frac{1}{\sqrt{x-[x]}}$ [Domain $(f)=R-Z$]

(iv) $f(x)+\frac{1}{\sqrt{x+[x]}}$ [Domain $(f)=(0,\infty)$]

50. Find the domain of definition of the function $f(x)$ given by $f(x)=\frac{1}{\log_{10}(1-x)}+\sqrt{x+2}$.

[Domain $(f)=(-\infty,0)\cup(0,1)\cap[-2,\infty)=[-2,0)\cup(0,1]$]

51. Find the range of each of the following functions:

(i) $f(x) = |x-3|$ [Range $(f) = [0, \infty)$]

(ii) $f(x) = 1-[x-2]$ [Range $(f) = (-\infty, 1]$]

(iii) $f(x) = \dfrac{|x-4|}{x-4}$ [Range $(f) = \{-1, 1\}$]

52. Find the domain and range of each of the following functions given by

(i) $f(x) = \dfrac{1}{\sqrt{x-[x]}}$ [Domain $(f) = R - Z$ Range $(f) = (1, \infty)$]

(ii) $f(x) = 1 - |x-3|$ [Domain $(f) = R$ Range $(f) = (-\infty, 1]$]

53. Find the domain of the real function $f(x)$ defined by $f(x) = \sqrt{\dfrac{1-|x|}{2-|x|}}$

[$(f) = (-\infty, -2) \cup (2, \infty) \cup [-1, 1]$]

54. Find the domain and range of the functions

(i) f (x) = $\sqrt{\dfrac{(2x+1)(2x+3)}{x+1}}$

[Domain : $\left[\dfrac{-3}{2},\ 1\right) \cup \left[\dfrac{-1}{2},\ \infty\right]$, Range : set of all positive real numbers]

(ii) f (x) = $\dfrac{1}{\sqrt{x^2-1}}$ [Domain : R – [– 1, 1], Range : set of all positive real numbers]

(iii) $\dfrac{1}{\sqrt{x-3}}$ [Domain : (3, ∞), Range : set of real positive numbers.]

55. Draw the graphs of the following functions in the given domains:

y = |x + 1|

56. (i) If f (x) = $\dfrac{1+x}{1-x}$, x ≠ 1, show that (fofofof)x = x.

(ii) If f (x) = $4x^2$ and g (x) = $\dfrac{\sqrt{x}}{2}$, then find (fog)(x) and (gof) (x). [(ii) (fog) (x) = x = (gof) (x)]

57. A function is of the form f (x) = ax + b. If (fof) (x) = 4x – 9, find the values of a and b.

[a = 2, b = – 3, or a = – 2, b = 9.]

58. If y = f (x) = $\dfrac{x+2}{x-1}$, show that x = f (y). Also find fof (2). [fof (2) = 2]

59. Given the function f (x) = $\dfrac{a^x + a^{-x}}{2}$, a > 0, show that f (x + y) + f (x – y) = 2 f (x) f (y).

60. Let $f:[0, 1] \to [0, 1]$ be defined by $f(x) = \dfrac{1-x}{1+x}, 0 \le x \le 1$ and let $g:[0, 1] \to [0, 1]$ be defined by $g(x) = 4x(1-x),\ 0 \le x \le 1$. Determine the composite function fog and gof.

COMPETITIVE AID

1. ***Conditional Identities***

If A + B + C = 180º, then

(i) sin 2A + sin 2B + sin 2C = 4sin A sin B sin C.

(ii) cos 2A + cos 2B + cos 2C = –1 – 4 cos A cos B Cos C

(iii) $\sin A + \sin B + \sin C = 4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$

(iv) $\cos A + \cos B + \cos C = 1 + 4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$

(v) tan A + tan B + tan C = tan A tan B tan C

(vi) $\tan\frac{A}{2}\tan\frac{B}{2} + \tan\frac{B}{2}\tan\frac{C}{2} + \tan\frac{C}{2}\tan\frac{A}{2} = 1$

(vii) cot A cot B + cot B cot C + cot C cot A = 1

2. (a) If k be any integer, then

$\cos(2k\pi + \theta) = \cos\theta$

$\sin(2k\pi + \theta) = \sin\theta$

$\tan(k\pi + \theta) = \tan\theta$

(b) A function f(x) is said to be periodic if f(x + p) = f(x), where $p \neq 0$ is a smallest +ve real number called period of the function.

For Example : cos θ, sin θ are the periodic functions, each of period 2π & tan θ is the periodic function of period π.

Some important results on periodic functions

(i) If f(x) is a periodic function with period T and a,b ∈ R such that a > 0 then f(ax + b) is periodic with period T/|a| For example sin x is periodic with period 2π Therefore sin(3x + 2) is periodic with 2π/3

(ii) If $f_1(x), f_2(x), f_3(x)$ are periodic functions with periods T_1, T_2 and T_3 respectively then $a_1f_1(x) + a_2f_2(x) + a_3f_3(x)$ is a periodic function with period equal to LCM of T_1,T_2 and T_3 where a_1,a_2 and a_3 are real numbers For example period of $\sin\left(\frac{2\pi x}{3}\right) + \cos\left(\frac{\pi x}{2}\right)$ is the LCM of 3 and 4 and i.e 12

This is not a universal rule. For Example If f(x) = |sin x| + |cos x|. The period of |sinx| is π and the period of |cos x| is πand if we take L.C.M then we are expecting its period is π, but actually its period is π/2. To remove this confusion if φ(x) is of the form f (sin x) + f (cos x), f (tan x) + f(cot x), f (sec x) + f (cosec x) and f is an even function then we have to divide a period by 2.This can also apply if they are in product form.

3. *T-Ratios of the Sum of three or More Angles*

(a) $\sin(A+B+C) = \sin A \cos B \cos C + \cos A \sin B \cos C + \cos A \cos B \sin C - \sin A \sin B \sin C$

or $\sin(A+B+C) = \cos A \cos B \cos C (\tan A + \tan B + \tan C - \tan A \tan B \tan C)$

(b) $\cos(A+B+C) = \cos A \cos B \cos C - \sin A \sin B \cos C - \sin A \cos B \sin C - \cos A \sin B \sin C$

$\cos(A+B+C) = \cos A \cos B \cos C (1 - \tan A \tan B - \tan B \tan C - \tan C \tan A)$

(c) $\tan(A+B+C) = \dfrac{\tan A + \tan B + \tan C - \tan A \tan B \tan C}{1 - \tan A \tan B - \tan B \tan C - \tan C \tan A}$

(d) $\sin(A_1 + A_2 + \ldots + A_n) = \cos A_1 \cos A_2 \ldots \cos A_n (S_1 - S_3 + S_5 - S_7 + \ldots)$

(e) $\cos(A_1 + A_2 + \ldots + A_n) = \cos A_1 \cos A_2 \ldots \cos A_n (1 - S_2 + S_4 - S_6 + \ldots)$

(f) $\tan(A_1 + A_2 + \ldots A_n) = \dfrac{S_1 - S_3 + S_5 - S_7 + \ldots\ldots}{1 - S_2 + S_4 - S_6 + \ldots\ldots}$,

where $S_1 = \tan A_1 + \tan A_2 + \ldots + \tan A_n$ = the sum of the tangents of the separate angles,

$S_2 = \tan A_1 \tan A_2 + \tan A_1 \tan A_3 + \ldots$ = the sum of the tangents taken two at a time,

$S_3 = \tan A_1 \tan A_2 \tan A_3 + \tan A_2 \tan A_3 \tan A_4 + \ldots$

= the sum of the tangents taken three at a time, and so on.

If $A_1 = A_2 = \ldots\ldots = A_n = A$, then

$S_1 = n \tan A$, $S_2 = {}^nC_2 \tan^2 A$, $S_3 = {}^nC_3 \tan^3 A, \ldots$ Therefore

(g) $\sin nA = \cos^n A({}^nC_1 \tan A - {}^nC_3 \tan^3 A + {}^nC_5 \tan^5 A - \ldots)$

(h) $\cos nA = \cos^n A(1 - {}^nC_2 \tan^2 A + {}^nC_4 \tan^4 A + \ldots)$

(i) $\tan nA = \dfrac{{}^nC_1 \tan A - {}^nC_3 \tan^3 A + {}^nC_5 \tan^5 A \ldots\ldots}{1 - {}^nC_2 \tan^2 A + {}^nC_4 \tan^4 A - {}^nC_6 \tan^6 A + \ldots}$

(j) $\sin nA + \cos nA = \cos^n A(1 + {}^nC_1 \tan A - {}^nC_2 \tan^2 A - {}^nC_3 \tan^3 A + {}^nC_4 \tan^4 A + {}^nC_5 \tan^5 A - {}^nC_6 \tan^6 A - {}^nC_7 \tan^7 A + \ldots)$

(k) $\sin nA - \cos nA = \cos^n A(-1 + {}^nC_1 \tan A + {}^nC_2 \tan^2 A - {}^nC_3 \tan^3 A - {}^nC_4 \tan^4 A + {}^nC_5 \tan^5 A + {}^nC_6 \tan^6 A \ldots)$

(*l*) $\sin \alpha + \sin(\alpha + \beta) + \sin(\alpha + 2\beta) + \ldots + \sin(\alpha + (n-1)\beta)$

$$= \sin\left(\frac{1st\angle + last\angle}{2}\right)\frac{\sin\frac{n\beta}{2}}{\sin\left(\frac{\beta}{2}\right)} = \frac{\sin\left(\alpha + (n-1)\frac{\beta}{2}\right)\sin\left(\frac{n\beta}{2}\right)}{\sin \beta/2}$$

$\cos \alpha + \cos(\alpha + \beta) + \cos(\alpha + 2\beta) + \ldots + \cos(\alpha + (n-1)\beta)$

$$= \cos\left(\frac{1st\angle + last\angle}{2}\right)\frac{\sin\frac{n\beta}{2}}{\sin\left(\frac{\beta}{2}\right)} = \frac{\cos\left(\alpha + (n-1)\frac{\beta}{2}\right)\sin\left(\frac{n\beta}{2}\right)}{\sin \beta/2}$$

WORKSHEET-I

Problems based on angles and their measurements

1. Find the degree measure corresponding to the following radian measures :

(i) $\left(\frac{9\pi}{5}\right)^c$

(ii) $(-4)^c$

(iii) $\left(\frac{7\pi}{6}\right)^c$ [(i) 324° (ii) –229° 5′ 54″ approx. (iii) 210°]

2. Find the radian measure corresponding to the following degree measures

(i) 300° [$\left(\frac{5\pi}{3}\right)^c$]

(ii) 125° 30′ [$\left(\frac{251\pi}{360}\right)^c$]

(iii) 7°30′ [$\left(\frac{\pi}{24}\right)^c$]

3. Express 45° 20′ 10′′ in Radian measure [0.79 radian]

4. The angles of triangle are in the ratio 3 : 4 : 5. Find the smallest angle in degrees and greatest angle in radians. [45°, $\left(\frac{5\pi}{12}\right)^C$]

5. The angles of a triangle are in A.P and the number of grades in the least is to number of radians in the greatest is 40 : π, find the angles in degrees.

6. Find the distance from the eye at which a coin of 2 cm diameter should be held so as to conceal the full moon whose angular diameter is 31′. [20°, 60°, 100°]

7. The perimeter of a certain sector of a circle is equal to the length of the arc of the semicircle having the same radius express the angle of the sector in degrees, minutes and seconds.
[2.217 m]

8. The minute hand of the clock is 10 cm long. How far does the tip of the hand move in 20 minutes?
[65° 24′ 30.4″]

9. A wire 121 cm long is bent so as to lie along the arc of a circle of radius 180 cm. Find in degrees the angle subtended at the centre by the arc.
[$\frac{20\pi}{3}$ cm]

6. 10.A man running along a circular track at the rate of 10 miles per hour traverses in 36 seconds, an arc which subtends an angle 56° at the centre, find the diameter of the circle. [38° 30′]

11. Find the angle between the hour hand and the minute hand in circular measure at half past 4.
[0.204 miles]

12. The number of sides of two regular polygons are in the ratio 5 : 4 and the difference between their each interior angle is 9°, find the number of sides of the two polygons. [45°]

13. The angle in one regular polygon is to that in another as 3 : 2 and the number of sides in the first is twice that in the second. Determine the number of sides of the two polygons.

$(1 + \tan\alpha \tan\beta)^2 + (\tan\alpha - \tan\beta)^2 = \sec^2\alpha \sec^2\beta$. [10 and 8]

14. $\dfrac{\tan A}{1-\cot A}+\dfrac{\cot A}{1-\tan A}$ = sec A cosec A + 1. [4 and 8]

Prove the following identities (15 to 23)

15. $2\sec^2\theta - \sec^4\theta - 2\,\text{cosec}^2\theta + \text{cosec}^4\theta = \cot^4\theta - \tan^4\theta$.

16. $\sin^2 A \cos^2 B - \cos^2 A \sin^2 B = \sin^2 A - \sin^2 B$.

17. $\dfrac{\text{cosec}\,\theta}{\text{cosec}\,\theta-1}+\dfrac{\text{cosec}\,\theta}{\text{cosec}\,\theta+1}=2\sec^2\theta$.

18. (secA – cosecA) (1 + tanA + cot A) = tan A secA – cot A cosec A.

19. $(\tan A + \text{cosec}\,B)^2 - (\cot B - \sec A)^2 = 2\tan A \cot B(\text{cosec}\,A + \sec B)$.

20. $\dfrac{2\sin\theta\tan\theta(1-\tan\theta)+2\sin\theta\sec^2\theta}{(1+\tan\theta)^2}=\dfrac{2\sin\theta}{1+\tan\theta}$.

21. $\dfrac{\cot^2\theta(\sec\theta-1)}{1+\sin\theta}=\sec^2\theta.\dfrac{1-\sin\theta}{1+\sec\theta}$

22. $(\text{cosec}\theta - \sec\theta)(\cot\theta - \tan\theta) = (\text{cosec}\theta + \sec\theta)(\sec\theta\,\text{cosec}\theta - 2)$

23. $(\tan\theta + \text{cosec}\phi)^2 - (\cot\phi - \sec\theta)^2 = 2\tan\theta\cot\phi(\text{cosec}\,\theta + \sec\phi)$

Problems based on elimination of θ (24 to 33)

24. If $\text{cosec}\,\theta - \sin\theta = a^3$, $\sec\theta - \cos\theta = b^3$, then prove that $a^2b^2(a^2 + b^2) = 1$

25. If $\cot\theta(1 + \sin\theta) = 4m$ and $\cot\theta(1 - \sin\theta) = 4n$, then prove that $(m^2 - n^2)^2 = mn$

28. if sinθ and cosθ are the roots of $ax^2 - bx + c = 0$ show that $a^2 - b^2 + 2ac = 0$

29. if cosx + sinx = $\sqrt{2}$ cosx, prove that cosx – sinx = $\pm\sqrt{2}$ sinx

30. If $\dfrac{\cos\alpha}{\cos\beta}=a,\dfrac{\sin\alpha}{\sin\beta}=b$, then prove that $(a^2 - b^2)\sin^2\beta = a^2 - 1$

31. If $\dfrac{ax}{\cos\theta}+\dfrac{by}{\sin\theta}=(a^2-b^2)$ *and* $\dfrac{ax\sin\theta}{\cos^2\theta}-\dfrac{by\cos\theta}{\sin^2\theta}$ = 0, prove that: $(ax)^{2/3} + (by)^{2/3} = (a^2 - b^2)^{2/3}$

32. If $c\cos^3\theta + 3c\cos\theta\sin^2\theta = m$, $c\sin^3\theta + 3c\cos^2\theta\sin\theta = n$ then prove that $(m + n)^{2/3} + (m - n)^{2/3} = 2c^{2/3}$.

33. Eliminate θ from the relations $a\sec\theta = 1 - b\tan\theta$, $a^2\sec^2\theta = 5 + b^2\tan^2\theta$ [$a^2b^2 + 4a^2 = 9b^2$]

34. If $\sin\alpha + \text{cosec}\,\alpha = 2$, prove that $\sin^n\alpha + \text{cosec}^n\alpha = 2$

35. If (1 – sinA) (1 – sin B) (1 – sinC) = (1 + sinA) (1 + sinB) (1 + sinC) prove that each side is ± cos A cos B cos C

36. If sec θ + tan θ = 4, find sec θ and tan θ. [secθ = $\dfrac{17}{8}$, tanθ = $\dfrac{15}{8}$]

37. If $\sin\theta + \sin^2\theta = 1$ prove that $\cos^{12}\theta + 3\cos^{10}\theta + 3\cos^8\theta + \cos^6\theta - 1 = 0$.

38. If $\sin\theta + \sin^2\theta + \sin^3\theta = 1$, then prove that $\cos^6\theta - 4\cos^4\theta + 8\cos^2\theta = 4$

PROBLEMS BASED ON ALLIED ANGLES

39. Find the values of each of the following trigonometric ratios :

(i) $\sin 510^o$ $[\frac{1}{2}]$

(ii) $\operatorname{cosec}(-405^o)$ $[-\frac{1}{\sqrt{2}}]$

(iii) $\tan\frac{11\pi}{6}$ $[-\frac{1}{\sqrt{3}}]$

(iv) $\cos 855^o$. $[-\frac{1}{\sqrt{2}}]$

40. Show that $\tan 9^o . \tan 27^o . \tan 45^o . \tan 63^o . \tan 81^o = 1$.

41. Prove that $\sec\left(\frac{3\pi}{2}-\theta\right)\sec\left(\theta-\frac{5\pi}{2}\right)+\tan\left(\frac{5\pi}{2}+\theta\right)\tan\left(\theta-\frac{3\pi}{2}\right) = -1$.

42. In a ΔABC, prove that

(i) $\cos(A + B) + \cos C = 0$

(ii) $\tan\frac{A+B}{2} = \cot\frac{C}{2}$

43. If ABCD is a cyclic quadrilateral then prove that $\cos A + \cos B + \cos C + \cos D = 0$.

44. Find the values of other five trigonometric functions in each of the following problems :

(i) $\sin\theta = \frac{3}{5}$, θ in quadrant I

$[\cos\theta = \frac{4}{5}, \tan\theta = \frac{3}{4}, \cot\theta = \frac{4}{3}, \sec\theta = \frac{5}{4}, \operatorname{cosec}\theta = \frac{5}{3}]$

(ii) $\cos\theta = -\frac{1}{2}$, θ in quadrant II

$[\sin\theta = \frac{\sqrt{3}}{2}, \tan\theta = -\sqrt{3}, \cot\theta = -\frac{1}{\sqrt{3}}, \sec\theta = -2, \operatorname{cosec}\theta = \frac{2}{\sqrt{3}}]$

46. If $\sin\theta = \frac{3}{5}$, $\tan\phi = \frac{1}{2}$ and $\frac{\pi}{2} < \theta < \pi < \phi < \frac{3\pi}{2}$ find the value of $8\tan\theta - \sqrt{5}\sec\phi$. $[-\frac{7}{2}]$

47. Verify that, $\cos 24^o + \cos 55^o + \cos 125^o + \cos 204^o + \cos 300^o = \frac{1}{2}$

48. Find the value of $\sin^2\frac{\pi}{18} + \sin^2\frac{\pi}{9} + \sin^2\frac{7\pi}{18} + \sin^2\frac{4\pi}{9}$. [2]

49. If $\cos A = -\frac{24}{25}$ and $\cos B = \frac{3}{5}$ where $\pi < A < \frac{3\pi}{2}$ and $\frac{3\pi}{2} < B < 2\pi$, then find the value of sin (A + B). [$\frac{3}{5}$]

50. Find x from the equation :

$$x \cot(90^o + \theta) + \tan(90^o + \theta) \sin\theta + \text{cosec}\,(90^o + \theta) = 0$$

[$x = \sin\theta$]

WORKSHEET-III

T – ratios of compounded angles

1. If $\sin\alpha \sin\beta - \cos\alpha \cos\beta + 1 = 0$ prove that $\cot\alpha \tan\beta + 1 = 0$
2. If $\tan\beta = \dfrac{n\sin\alpha\cos\alpha}{1-n\sin^2\alpha}$, prove that $\tan(\alpha-\beta) = (1-n)\tan\alpha$
3. Prove that $\dfrac{1}{\tan 3A - \tan A} - \dfrac{1}{\cot 3A - \cot A} = \cot 2A$
4. Prove that $\dfrac{\tan^2 2\theta - \tan^2\theta}{1-\tan^2 2\theta \tan^2\theta} = \tan 3\theta \tan\theta$
5. If $\tan B = \dfrac{2\sin A \sin C}{\sin(A+C)}$, Prove that cot A, cot B, cot C are in AP
6. If $2\tan\beta + \cot\beta = \tan\alpha$, prove that $\cot\beta = 2\tan(\alpha-\beta)$
7. If $A + B = \dfrac{5\pi}{4}$, prove that $\dfrac{\cot A}{1+\cot A}\cdot\dfrac{\cot B}{1+\cot B} = \dfrac{1}{2}$
8. If $m\tan\left(\theta - \dfrac{\pi}{6}\right) = n\tan(\theta + 2\pi/3)$, prove that $\cos 2\theta = \dfrac{m+n}{2(m-n)}$
9. If $\tan(A-B) = x$ and $\tan(A+B) = y$, prove that $\tan 2B = \dfrac{y-x}{1+xy}$
10. If $\tan x + \tan(x+\pi/3) + \tan(x+2\pi/3) = 3$ then prove that $\dfrac{3\tan x - \tan^3 x}{1-3\tan^2 x} = 1$
11. If $\tan\alpha = \sqrt{a\lambda}$, $\tan\beta = \sqrt{\dfrac{\lambda}{a}}$ and $\tan\gamma = \sqrt{\dfrac{\lambda}{a^3}}$, where $\lambda = 1 - a - a^2$, prove that $\alpha + \beta = \gamma$
12. Prove that $\dfrac{\tan(A+B)}{\cot(A-B)} = \dfrac{\sin^2 A - \sin^2 B}{\cos^2 A - \sin^2 B}$
13. Prove that $\sin^2\left(\dfrac{\pi}{8} + \dfrac{A}{2}\right) - \sin^2\left(\dfrac{\pi}{8} - \dfrac{A}{2}\right) = \left(\dfrac{1}{\sqrt{2}}\right)\sin A$
14. Prove that $\tan 3A \tan 2A \tan A = \tan 3A - \tan 2A - \tan A$
15. Prove that $\dfrac{\cos 9^0 + \sin 9^0}{\cos 9^0 - \sin 9^0} = \tan 54^0$
16. If θ lies in the first quadrant and $\cos\theta = 8/17$ then prove that

    $\cos\left(\dfrac{\pi}{6}+\theta\right) + \cos\left(\dfrac{\pi}{4}-\theta\right) + \cos\left(\dfrac{2\pi}{3}-\theta\right) = \left(\dfrac{\sqrt{3}-1}{2} + \dfrac{1}{\sqrt{2}}\right)\dfrac{23}{17}$
17. If three angles A, B, C are in AP prove that $\cot B = \dfrac{\sin A - \sin C}{\cos C - \cos A}$
18. If in a triangle ABC, $\sin A \cos B = 1/4$, and $3\tan A = \tan B$ then prove that triangle is right angled

19. If $\sin\alpha + \sin\beta = a$ and $\cos\alpha + \cos\beta = b$, show that

    (i) $\cos(\alpha+\beta) = \dfrac{b^2 - a^2}{b^2 + a^2}$ (ii) $\sin(\alpha+\beta) = \dfrac{2ab}{a^2+b^2}$
20. If $\tan(\pi\cos\theta) = \cot(\pi\sin\theta)$, prove that $\cos(\theta - \pi/4) = \pm 1/2\sqrt{2}$
21. If α and β are solutions of $a\cos\theta + b\sin\theta = c$ then show that

(i) $\cos(\alpha + \beta) = (a^2 - b^2)/(a^2 + b^2)$

(ii) $\cos(\alpha - \beta) = [2c^2 - (a^2 + b^2)]/(a^2 + b^2)$

22. If α and β are solutions of the equation $a \tan\theta + b \sec\theta = c$ then show that

$\tan(\alpha + \beta) = \dfrac{2ac}{a^2 - c^2}$

23. Show that the maximum and minimum values of $8 \cos\theta - 15 \sin\theta$ lies between -17 and 17

24. Prove that $5 \cos\theta + 3 \cos(\theta + \pi/3) + 3$ lies between -4 and 10

25. If $\sin x + \sin y = 3(\cos y - \cos x)$ then prove that $\sin 3x + \sin 3y = 0$

26. If $\alpha + \beta = \dfrac{\pi}{2}$ and $\beta + \gamma = \alpha$ then prove that $\tan\alpha = \tan\beta + 2\tan\gamma$

27. If $\cos(x - y) + \cos(y - z) + \cos(z - x) = -\dfrac{3}{2}$, then prove that

$\cos x + \cos y + \cos z = 0 = \sin x + \sin y + \sin z$

28. If $x \sin\theta = y \sin(\theta + 2\pi/3) = z \sin(\theta + 4\pi/3)$ then prove that $\sum xy = 0$

29. If $\cos(\alpha + \beta) = \dfrac{4}{5}$, $\sin(\alpha - \beta) = \dfrac{5}{13}$ and α, β lie between 0 and $\dfrac{\pi}{4}$ then prove that

$\tan 2\alpha = \dfrac{56}{33}$

30. Prove that $\tan\alpha + \tan\beta + \tan\gamma - \tan\alpha \tan\beta \tan\gamma = \dfrac{\sin(\alpha + \beta + \gamma)}{\cos\alpha \cos\beta \cos\gamma}$

WORKSHEET-IV

T – ratios of transformation formulae:

1. Prove that

(i) $\sin 38^0 + \sin 22^0 = \sin 82^0$

(ii) $\sin 40^0 + \sin 20^0 = \cos 10^0$

(iii) $\cos 55^0 + \cos 65^0 + \cos 175^0 = 0$

(iv) $\cos 20^0 + \cos 100^0 + \cos 140^0 = 0$

(v) $\sin 5\pi/18 - \cos 4\pi/9 = \sqrt{3} \sin \pi/9$

2. Prove that

(i) $\cos 3A + \cos 5A + \cos 7A + \cos 15A = 4 \cos 4A \cos 5A \cos 6A$

(ii) $\sin A + \sin 2A + \sin 4A + \sin 5A = 4 \cos A/2 \cos 3A/2 \sin 3A$

(iii) $\cos 20^0 \cos 100^0 + \cos 100^0 \cos 140^0 - \cos 140^0 \cos 200^0 = -3/4$

(iv) $\sin \theta/2 \sin 7\theta/2 + \sin 3\theta/2 \sin 11\theta/2 = \sin 2\theta \sin 5\theta$

3. Prove that

(i) $\cos 10^0 \cos 30^0 \cos 50^0 \cos 70^0 = 3/16$

(ii) $\cos 40^0 \cos 80^0 \cos 160^0 = -1/8$

(iii) $\tan 20^0 \tan 40^0 \tan 60^0 \tan 80^0 = 3$

(iv) $\sin 10^0 \sin 50^0 \sin 60^0 \sin 70^0 = \sqrt{3}/16$

4. Prove that

(i) $\dfrac{\sin A + \sin 3A + \sin 5A}{\cos A + \cos 3A + \cos 5A} = \tan 3A$

(ii) $\dfrac{\cos 3A + 2\cos 5A + \cos 7A}{\cos A + 2\cos 3A + \cos 5A} = \dfrac{\cos 5A}{\cos 3A}$

(iii) $\dfrac{\sin 3A + \sin 5A + \sin 7A + \sin 9A}{\cos 3A + \cos 5A + \cos 7A + \cos 9A} = \tan 6A$

(iv) $\dfrac{\sin 5A - \sin 7A + \sin 8A - \sin 4A}{\cos 4A + \cos 7A - \cos 5A - \cos 8A} = \cot 6A$

(v) $\dfrac{\sin 11A \sin A + \sin 7A \sin 3A}{\cos 11A \sin A + \cos 7A \sin 3A} = \tan 8A$

(vi) $\dfrac{\sin(\theta+\varphi) - 2\sin\theta + \sin(\theta-\varphi)}{\cos(\theta+\varphi) - 2\cos\theta + \cos(\theta-\varphi)} = \tan\theta$

5. Prove that

(i) $\sin \alpha + \sin \beta + \sin \gamma - \sin(\alpha + \beta + \gamma) = 4\sin\left(\dfrac{\alpha+\beta}{2}\right)\sin\left(\dfrac{\beta+\gamma}{2}\right)\sin\left(\dfrac{\gamma+\alpha}{2}\right)$

(ii) $\cos (A + B + C) + \cos(A - B + C) + \cos(A + B - C) + \cos(-A + B + C)$
$= 4 \cos A \cos B \cos C$

6. $\cos A + \cos B = 1/2$, $\sin A + \sin B = 1/4$, prove that $\tan\left(\frac{A+B}{2}\right) = \frac{1}{2}$

7. If cosec A + secA = cosec B + sec B , then prove that tan A tan B = $\cot\left(\frac{A+B}{2}\right)$

8. If $\sin 2A = \lambda \sin 2B$, prove that $\frac{\tan(A+B)}{\tan(A-B)} = \frac{\lambda+1}{\lambda-1}$

9. If $\cos\alpha = \lambda\cos(\alpha - 2\beta)$, prove that $\tan(\alpha-\beta)\tan\beta = \frac{1-\lambda}{1+\lambda}$

10. Prove that

    (i) $\cos^2(A - B) + \cos^2 B - 2\cos A\cos B\cos(A - B) = \sin^2 A$

    (ii) $\sin^2 A + \sin^2(A - B) - 2\sin A\cos B\sin(A - B) = \sin^2 B$

    (iii) $\cos^2 A + \cos^2 B - 2\cos A\cos B\cos(A + B) = \sin^2(A + B)$

11. Prove that

    (i) $\sin 50^0 - \sin 70^0 + \sin 10^0 = 0$

    (ii) $\sin 10^0 + \sin 20^0 + \sin 40^0 + \sin 50^0 - \sin 70^0 - \sin 80^0 = 0$

12. Prove that

    (i) $\frac{\cos(A+B+C)+\cos(-A+B+C)+\cos(A-B+C)+\cos(A+B-C)}{\sin(A+B+C)+\sin(-A+B+C)+\sin(A-B+C)-\sin(A+B-C)} = \cot B$

    (ii) $\sin(B - C)\cos(A - D) + \sin(C - A)\cos(B - D) + \sin(A - B)\cos(C - D) = 0$

13. Prove that $\frac{\sin^2 A - \sin^2 B}{\sin A\cos A - \sin B\cos B} = \tan(A+B)$

WORKSHEET-V

T – ratios of multiple and sub multiple angles:

1. Prove the following identities :

   (i) $\tan(\pi/4 + \theta) - \tan(\pi/4 - \theta) = 2\tan 2\theta$

   (ii) $\tan A + \cot A = 2\,\text{cosec}\,2A$

   (iii) $(\tan\alpha \sec^2\alpha + \cot\alpha\,\text{cosec}^2\alpha) - (\tan\alpha + \cot\alpha)$

   $= 2\,\text{cosec}\,2\alpha\,(\sec^2\alpha + \text{cosec}^2\alpha - 3)$

   (iv) $\cot A - \tan A = 2\cot 2A$

   (v) $\tan A + 2\tan 2A + 4\tan 4A + 8\cot 8A = \cot A$

   (vi) $\text{cosec}\,A - 2\cot 2A\cos A = 2\sin A$

   (vii) $\text{cosec}\,2A - \cot 2A = \tan A$

2. Prove the following identities :

   (i) $\dfrac{1-\cos A}{\sin A} = \tan\dfrac{A}{2}$

   (ii) $\dfrac{\cos 2\theta}{1+\sin 2\theta} = \tan\left(\dfrac{\pi}{4} - \theta\right)$

   (iii) $2\sin A\cos^3 A - 2\sin^3 A\cos A = (1/2)\sin 4A$

   (iv) $\dfrac{1+\sin 2\theta - \cos 2\theta}{1+\sin 2\theta + \cos 2\theta} = \tan\theta$

   (v) $\sqrt{2+\sqrt{2+\sqrt{2+2\cos 8\theta}}} = 2\cos\theta$

   (vi) $\sin A\sin(60^0 - A)\sin(60^0 + A) = (1/4)\sin 3A$

3. Prove the following identities :

   (i) $2\cos x - \cos 3x - \cos 5x = 16\cos^3 x\sin^2 x$

   (ii) $1 + \cos 2x + \cos 4x + \cos 6x = 4\cos x\cos 2x\cos 3x$

4. Prove that $(\cos\alpha + \cos\beta)^2 + (\sin\alpha + \sin\beta)^2 = 4\cos^2\left(\dfrac{\alpha-\beta}{2}\right)$

5. Prove that $\cos\phi = \dfrac{\cos\theta - e}{1 - e\cos\theta}$ *if* $\tan\dfrac{\theta}{2} = \sqrt{\dfrac{1-e}{1+e}}\tan\dfrac{\varphi}{2}$

6. If $\cos\theta = \dfrac{2\cos\varphi - 1}{2 - \cos\varphi}$, prove that $\tan\theta/2 = \pm\sqrt{3}\tan\phi/2$ and hence show that $\sin\varphi = \dfrac{\sqrt{3}\sin\theta}{2+\cos\theta}$

7. If $\tan\beta = \cos\theta\tan\alpha$ then prove that $\sin(\alpha - \beta) = \tan^2\theta/2\,\sin(\alpha + \beta)$

8. If $\tan^2\theta = 2\tan^2\phi + 1$ then prove that $\cos 2\theta + \sin^2\phi = 0$

9. If $\sin\theta$ is GM of $\sin\phi$ and $\cos\phi$ then prove that $\cos 2\theta = 2\cos^2(\pi/4 + \phi)$

10. Prove the following identities :

(i) $\cos 2\alpha = 2\sin^2\beta + 4\cos(\alpha+\beta)\sin\alpha\sin\beta + \cos 2(\alpha+\beta)$

(ii) $\cos^2 A + \cos^2 B - 2\cos A\cos B\cos(A+B) = \sin^2(A+B)$

(iii) $\sin^2(\theta+A) + \sin^2(\theta+B) - 2\cos(A-B)\sin(\theta+A)\sin(\theta+B) = \sin^2(A-B)$

11. Prove that :

(i) $1 + \cos 56^0 \cos 58^0 - \cos 66^0 = 4\cos 28^0 \cos 29^0 \sin 33^0$

(ii) $\cos^2 73^0 + \cos^2 47^0 + \cos 73^0 \cos 47^0 = 3/4$

(iii) $\sin^2 12^0 + \sin^2 21^0 + \sin^2 39^0 + \sin^2 48^0 = 1 + \sin^2 9^0 + \sin^2 18^0$

12. Prove that :

(i) $\dfrac{\sec 8\theta - 1}{\sec 4\theta - 1} = \dfrac{\tan 8\theta}{\tan 2\theta}$

(ii) $\tan A \sec 4A + \tan 4A = \tan A + \tan 4A \sec 2A$

13. Prove the following identities :

(i) $\cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$

(ii) $\sin 5\theta = 5\cos^4\theta\sin\theta - 10\cos^2\theta\sin^3\theta + \sin^5\theta$

14. Prove the following identities :

(i) $\cos^3\theta + \cos^3(120^0 + \theta) + \cos^3(240^0 + \theta) = (3/4)\cos 3\theta$

(ii) $\cos^2 A + \cos^2(A + 120^0) + \cos^2(A - 120^=) = 3/2$

15. Prove that :

(i) $\cos 2\pi/7 + \cos 4\pi/7 + \cos 6\pi/7 = -1/2$

(ii) $\cos \pi/11 + \cos 3\pi/11 + \cos 5\pi/11 + \cos 7\pi/11 + \cos 9\pi/11 = 1/2$

16. Prove that :

(i) $\cos 2\pi/15 \cos 4\pi/15 \cos 8\pi/15 \cos 16\pi/15 = 1/16$

(ii) $\cos \pi/7 \cos 2\pi/7 \cos 4\pi/7 = -1/8$

(iii) $\cos \pi/33 \cos 2\pi/33 \cos 4\pi/33 \cos 8\pi/33 \cos 16\pi/33 = 1/32$

(iv) $\cos \pi/15 \cos 2\pi/15 \cos 3\pi/15 \cos 4\pi/15 \cos 5\pi/15 \cos 6\pi/15 \cos 7\pi/15 = 1/128$

(v) $\sin \pi/14 \sin 3\pi/14 \sin 5\pi/14 \sin 7\pi/14 \sin 9\pi/14 \sin 11\pi/14 \sin 13\pi/14 = 1/64$

17. If $\theta = \dfrac{\pi}{2^n + 1}$, prove that $2^n \cos\theta \cos 2\theta \cos 2^2\theta \ldots\ldots \cos 2^{n-1}\theta = 1$

18. Prove that :

    (i) $\sqrt{3}$ cosec 20^0 – sec 20^0 = 4

    (ii) 4 cos 20^0 – $\sqrt{3}$ cot 20^0 = 1

T – ratios of some important angles:

19. Prove that cos 6^0 cos 42^0 cos 66^0 cos 78^0 = 1/16

20. Prove that cot 7 ½ ^{0} = $\sqrt{2} + \sqrt{3} + \sqrt{4} + \sqrt{6}$

21. Show that

    a. (a) $\cot 22\frac{1}{2}^0 = \sqrt{2}+1$

    b. $\tan 11\frac{1}{4}^0 = \sqrt{4+2\sqrt{2}} - \sqrt{2} - 1$

22. Prove that tan9^0 – tan 27^0 – tan 63^0 + tan 81^0 = 4

23. Prove that 4 sin 27^0 = $(5 + \sqrt{5})^{1/2} - (3 - \sqrt{5})^{1/2}$

WORKSHEET-VI

1. Find the principle solutions of the following equations:

(i) $\tan x = \frac{1}{\sqrt{3}}$ [$\frac{\pi}{6}$ or $\frac{7\pi}{6}$]

(ii) $\sec x = 2$ [$\frac{\pi}{3}$ or $\frac{5\pi}{3}$]

(iii) $\cot x = -\sqrt{3}$ [$\frac{5\pi}{6}$ or $\frac{11\pi}{6}$]

(iv) $\text{cosec } x = -2$ [$\frac{7\pi}{6}$ or $\frac{11\pi}{6}$]

2. Find the general solution of the following equations :

(i) $\sin 2\theta = 0$ [$\theta = \frac{n\pi}{2},\ n \in I$]

(ii) $\tan 3x = 0$ [$x = \frac{n\pi}{3},\ n \in I$]

(iii) $\cos = \frac{3\theta}{2} = 0$ [$\theta = (2n+1)\frac{\pi}{3},\ n \in I$]

(iv) $\tan^2 7\theta = 0.$ [$\theta = \frac{n\pi}{7},\ n \in I$]

3. Find the general solution of the following equations :

(i) $\sin\theta = \frac{\sqrt{3}}{2}$ [$\theta = n\pi + (-1)^n \frac{\pi}{3},\ n \in I$]

(ii) $\cos\theta = -\frac{\sqrt{3}}{2}$ [$\theta = 2n\pi \pm \frac{5\pi}{6},\ n \in I$]

4. Find the general solutions of the following equation :

(i) $2(\cos^2\theta - \sin^2\theta) = 1$ [$\theta = n\pi \pm \frac{\pi}{6},\ n \in I$]

(ii) $\sec\theta = -\sqrt{2}$ [$\theta = 2n\pi \pm \frac{3\pi}{4},\ n \in I$]

5. Find the general solution of the following equations :

(i) $\sin\frac{\theta}{2}=-\frac{1}{2}$ [$\theta=2n\pi+(-1)^{n+1}\frac{\pi}{3},\ n\in I$]

(ii) $\cos\frac{5\theta}{2}=\frac{\sqrt{3}}{2}$ [$\theta=\frac{4n\pi}{5}\pm\frac{\pi}{15},\ n\in I$]

(iii) $\tan\frac{7\theta}{2}=1$ [$\theta=\frac{2n\pi}{7}+\frac{\pi}{14},\ n\in I$]

6. Find the general solutions of the following equations :

(i) $\tan 2x=-\cot\left(x+\frac{\pi}{6}\right)$ [$x=n\pi+\frac{2\pi}{3},\ n\in I$]

(ii) $\sec x=\sec(x+\pi)$ [$x=(2n+1)\frac{\pi}{2},\ n\in I$]

7. Solve the following equations :

(i) $\sin mx+\sin nx=0$ [$x=\frac{2p\pi}{m+n},\ p\in I$ or $x=\frac{(2q+1)\pi}{m-n},\ q\in I$]

(ii) $\sin x+\sin 3x+\sin 5x=0$ [$x=\frac{n\pi}{3},\ n\in I$ or $x=n\pi+\frac{\pi}{3},\ n\in I$]

(iii) $\sin 2x+\sin 4x=2\sin 3x$ [$x=\frac{n\pi}{3},\ n\in I$ or $x=2n\pi,\ n\in I$]

(iv) $\cos x+\cos 3x+\cos 5x+\cos 7x=0$

[$x=(2n+1)\frac{\pi}{8}$ or $x=(2n+1)\frac{\pi}{4}$ or $x=(2n+1)\frac{\pi}{2},\ n\in I$]

8. Solve the following equations :

(i) $\cos^2\theta=\frac{1}{4}$ [$\theta=n\pi\pm\frac{\pi}{3}, n\in I$]

(ii) $4\cos^2\theta=3$ [$\theta=n\pi\pm\frac{\pi}{6}, n\in I$]

(iii) $\sin^2\theta=\frac{1}{2}$ [$\theta=n\pi\pm\frac{\pi}{6}, n\in I$]

(iv) $\operatorname{cosec}^2\theta=\frac{4}{3}$ [$\theta=n\pi\pm\frac{\pi}{3}, n\in I$]

(v) $\tan^2\theta+\cot^2\theta=2$ [$\theta=n\pi\pm\frac{\pi}{4}, n\in I$]

(vi) $4\cot^2\theta = 3\ \text{cosec}^2\ \theta$ $\quad [\theta = n\pi \pm \frac{\pi}{6},\ n \in I\ ]$

(vii) $2\left(\sin^4 x + \cos^4 x\right) = 1$ $\quad [\theta = \frac{n\pi}{2} \pm \frac{\pi}{4},\ n \in I\ ]$

(viii) $2\sin^2 x + \sin^2 2x = 2$ $\quad [\ x = n\pi \pm \frac{\pi}{2}$ or $x = n\pi \pm \frac{\pi}{4},\ n \in I\ ]$

9. Solve the following equations :

(i) $\tan x + \tan 2x - \tan 3x = 0$ $\quad [\ x = \frac{n\pi}{3}$ or $x = \frac{n\pi}{2}\ ]$

(ii) $\tan x + \tan 2x + \tan 3x = 0$ $\quad [\ x = \frac{n\pi}{3}$ or $x = n\pi \pm \tan^{-1}\frac{1}{\sqrt{2}},\ n \in I\ ]$

(iii) $\tan x - 2\tan 2x + \tan 3x = 0$ $\quad [\ x = n\pi,\ n \in I\ ]$

(iv) $\sin 2x - 3\tan x \cos 2x = 0$ $\quad [\ x = n\pi$ or $x = n\pi \pm \frac{\pi}{6},\ n \in I\ ]$

(v) $\tan x + \tan 4x + \tan 7x = \tan x \tan 4x \tan 7x$ $\quad [\ x = \frac{n\pi}{12},\ n \in I\ ]$

(vi) $\tan\left(\frac{\pi}{4} + x\right) + \tan\left(\frac{\pi}{4} - x\right) = 4$ $\quad [\ x = n\pi \pm \frac{\pi}{6},\ n \in I\ ]$

10. Solve the following equations :

(i) $2\cos^2 x + 3\sin x = 0$ $\quad [\ n\pi + (-1)^n \frac{\pi}{6},\ n \in I\ ]$

(ii) $7\cos^2 x + 3\sin^2 x = 5$ $\quad [\ x = n\pi \pm \frac{\pi}{4},\ n \in I\ ]$

(iii) $4\sin^2 x + \sqrt{3} = 2\left(\sqrt{3} + 1\right)\sin x$ $\quad [\ x = n\pi + (-1)^n \frac{\pi}{6}\ \ n \in I,\ \ x = n\pi + (-1)^n \frac{\pi}{3}\ \ n \in I\ ]$

(iv) $2\cos^2 x + \cos\ 2x = 2$ $\quad [\ x = n\pi$ or $x = n\pi \pm \frac{\pi}{6},\ n \in I\ ]$

(v) $\cot^2 x + \frac{3}{\sin x} = -3$ $\quad [\ x = n\pi + (-1)^{n+1}\frac{\pi}{6}\ \ n \in I,\ \ x = n\pi + (-1)^{n+1}\frac{\pi}{2}\ \ n \in I\ ]$

(vi) $\tan^2 x + \left(1 - \sqrt{3}\right)\tan x - \sqrt{3} = 0$ $\quad [\ x = \frac{n\pi}{3} + \frac{\pi}{3}$ or $x = n\pi - \frac{\pi}{4}\ \ n \in I\ ]$

(vii) $\text{cosec}^2 2x = 1 - \cot 2x$ [$x = (2n+1)\frac{\pi}{4}$ or $x = \frac{n\pi}{2} - \frac{\pi}{8},\ n \in I$]

(viii) $\frac{\sin x}{\cos x} - \frac{\cos x}{\sin x} + \frac{1}{\cos x} = 0$ [$x = n\pi + (-1)^{n+1}\frac{\pi}{2}\ \ n \in I,\ \ x = n\pi + (-1)^n \frac{\pi}{6}\ \ n \in I$]

11. Solve the following equations :

(i) $\cos\theta - \sin\theta = -1$ [$\theta = (2n+1)\frac{\pi}{2}$ or $\theta = (2n-1)\pi,\ n \in I$]

(ii) $\sqrt{3}\cos x - \sin x = 1$ [$x = n\pi + (-1)^{n+1}\frac{\pi}{6} + \frac{\pi}{3},\ n \in I$]

(iii) $\cos x + \sin x = 1$ [$x = 2n\pi$ or $x = (4n+1)\frac{\pi}{2},\ n \in I$]

(iv) $2\sin x + \sqrt{3}\cos x = 1 + \sin x$ [$x = (4n+1)\frac{\pi}{2}$ or $x = 2n\pi - \frac{\pi}{6},\ n \in I$]

(v) $\sec x - \tan x = \sqrt{3}$ [$x = (4n+1)\frac{\pi}{2}$ or $x = 2n\pi - \frac{\pi}{6},\ n \in I$]

(vi) $\text{cosec}\,\theta + \cot\theta = \sqrt{3}$ [$\theta = 2n\pi + \frac{\pi}{3},\ n \in I$ or $\theta = (2n-1)\pi,\ n \in I$]

(vii) $\cos^2\theta - 2\cos\theta\sin\theta - \sin 2\theta = 1$ [$\theta = n\pi$ or $\theta = n\pi - \frac{\pi}{4},\ n \in I$]

WORKSHEET-VII

1. Solve for x

$\sin x + \sqrt{3}\cos x \geq 1$ $\qquad$ [$\left[2n\pi - \frac{\pi}{6}, 2n\pi + \frac{\pi}{2}\right], n \in I$]

2. Find the set of value of x for which

$\frac{\tan 3x - \tan 2x}{1 + \tan 3x \tan 2x} = 1$ $\qquad$ [ϕ]

3. Solve for x

$\sin x - 3\sin 2x + \sin 3x = \cos x - 3\cos 2x + \cos 3x$ $\qquad$ [$x = (4n+1)\pi/8, n \in I$]

4. Find the general solution of the equation

$\left[\cos(x/4) - 2\sin x\right]\sin x + \left[1 + \sin(x/4) - 2\cos x\right]\cos x = 0.$ $\qquad$ [$x = 2(4m+1)\pi \,\forall\, m \in I$]

5. Solve $2^{\sec^2 x}\sqrt{y^2 - y + \frac{1}{2}} \leq 1$ for real x and y. $\qquad$ [$x = n\pi, n \in I; y = 1/2$]

6. Find all the values of x lying in the interval $(-\pi, \pi)$ which satisfy the equation $8^{1+|\cos x|+\cos^2 x+|\cos^3 x|+...+\infty} = 4^3.$ $\qquad$ [$\pm\frac{\pi}{3}, \pm\frac{2\pi}{3}$]

7. Solve for x

$3^{\sin 2x + 2\cos^2 x} + 3^{1 - \sin 2x + 2\sin^2 x} = 28.$ $\qquad$ [$x = (2n+1)\pi/2, n\pi - \pi/4, n \in I$]

8. Solve $\sin x + \sin y = \sin(x+y), |x| + |y| = 1.$ $\qquad$ [$(\pm 1, 0)(0, \pm 1)(\pm 1/2, \mp 1/2)$]

9. Solve the equation $\tan^3 x - 1 + \frac{1}{\cos^2 x} - 3\cot\left(\frac{\pi}{2} - x\right) = 3.$ $\qquad$ [$x = n\pi + \frac{3\pi}{4}$ or $x = n\pi \pm \frac{\pi}{3}, n \in I$]

10. Solve the following equations for θ :

$r\sin\theta = 3, r = 4(1 + \sin\theta), 0 \leq \theta \leq 2\pi$ $\qquad$ [$\pi/6, 5\pi/6$]

11. Solve the equation :

$2(\cos x + \cos 2x) + (1 + 2\cos x)\sin 2x = 2\sin x, -\pi \leq x \leq \pi.$ $\qquad$ [$-\pi, -\pi/2, -\pi/3, \pi/3, \pi$]

WORKSHEET-VIII

Solve the following problems based on conditional trigonometric identities :

For Questions 1 to 10: If A + B + C = π, prove the following identities :

1. sin 2A + sin 2B + sin 2C = 4 sin A sin B sin C
2. cos 2A + cos 2B + cos 2C = – 1 – 4 cos A cos B cos C
3. sin A + sin B + sin C = 4 cos A/2 cos B/2 cos C/2
4. cos A + cos B + cos C = 1 + 4 sin A/2 sin B/2 sin C/2
5. $\cos^2 A + \cos^2 B + \cos^2 C = 1 - 2 \cos A \cos B \cos C$
6. $\sin^2 A + \sin^2 B + \sin^2 C = 2 + 2 \cos A \cos B \cos C$
7. cos A/2 + cos B/2 + cos C/2 = 4 cos{(B + C)/4} cos{(C + A)/4} cos{(A + B)/4}
8. tan A + tan B + tan C = tan A tan B tan C
9. cot A cot B + cot B cot C + cot C cot A = 1
10. cot(A/2) + cot(B/2) + cot(C/2) = 4 cot(A/2) cot (B/2) cot (C/2)
11. If x + y + z = xyz then prove that

$$\frac{2x}{1-x^2}+\frac{2y}{1-y^2}+\frac{2z}{1-z^2} = \frac{2x}{1-x^2}\frac{2y}{1-y^2}\frac{2z}{1-z^2}$$

12. If xy + yz +zx = 1, then prove that

$$\frac{x}{1+x^2}+\frac{y}{1+y^2}+\frac{z}{1+z^2}=\frac{2}{\left[(1+x^2)(1+y^2)(1+z^2)\right]^{1/2}}$$

13. If $\alpha = 2\pi/7$, show that $\tan\alpha \tan 2\alpha + \tan 2\alpha \tan 4\alpha + \tan 4\alpha \tan \alpha = -7$
14. If A + B + C = π, show that $\tan^2 A/2 + \tan^2 B/2 + \tan^2 C/2 \geq 1$
15. Prove that the triangle is equilateral if and only if tan A + tan B + tan C = $3\sqrt{3}$
16. If A + B + C = 2S, show that

    $\cos^2 S + \cos^2(S - A) + \cos^2(S - B) + \cos^2(S - C) = 2 + 2 \cos A \cos B \cos C$
17. A wheel makes 360 revolutions in one minute. Through how many radians does it turn in one second?
18. In a circle of diameter 40 cm, the length of a chord is 20 cm. Find the length of minor arc of the chord.
19. If in two circles, arcs of the same length subtend angles 60° and 75° at the centre, find the ratio of their radii.
20. sin2 x + 2 sin 4x + sin 6x = 4 cos2 x sin 4x
21. cot 4x (sin 5x + sin 3x) = cot x (sin 5x – sin
22. $\dfrac{\cos 4x+\cos 3x+\cos 2x}{\sin 4x+\sin 3x+\sin 2x}=\cot 3x$
23. $\dfrac{\sin x-\sin y}{\cos x+\cos y}=\tan\dfrac{x-y}{2}$
24. $\tan 4x=\dfrac{4\tan x(1-\tan^2 x)}{1-6\tan^2 x+\tan^4 x}$

WORKSHEET-VIII

1. If $\cos\theta = \dfrac{\cos\alpha - \cos\beta}{1 - \cos\alpha\cos\beta}$, Prove that $\tan\theta/2 = \pm \tan\alpha/2 \cot\beta/2$

2. If $\sec(\phi - \alpha)$, $\sec\phi$, $\sec(\phi + \alpha)$ are in AP then prove that $\cos\phi = \pm\sqrt{2}\cos\alpha/2$

3. If $\tan\dfrac{x-y}{2}$, $\tan z$, $\tan\dfrac{x+y}{2}$ are in GP , prove that $\cos x = \cos y \cos 2z$

4. Prove that :

    (i) $\tan\theta\tan(\theta + 60^0) + \tan\theta\tan(\theta - 60^0) + \tan(\theta + 60^0)\tan(\theta - 60^0) = -3$

    (ii) $\cot 16^0 \cot 44^0 + \cot 44^0 \cot 76^0 - \cot 76^0 \cot 16^0 = 3$

    (iii) $\dfrac{3 + \cot 76^0 \cot 16^0}{\cot 76^0 + \cot 16^0} = \cot 44^0$

    (iv) $\cot A \cot 2A + \cot 2A \cot 3A + 2 = \cot A(\cot A - \cot 3A)$

    (v) $\tan A + \tan(60^0 + A) - \tan(60^0 - A) = 3\tan 3A$

    (vi) $\cot\alpha + \cot(60^0 + \alpha) - \cot(60^0 - \alpha) = 3\cot 3\alpha$

14. Prove that $\dfrac{\cos 3\theta + \cos 3\varphi}{2\cos(\theta - \varphi) - 1} = (\cos\theta + \cos\phi)\cos(\theta + \phi) - (\sin\theta + \sin\phi)\sin(\theta + \phi)$

15. If $a\cos 2\theta + b\sin 2\theta = c$ has α and β as its solutions, then prove that $\tan(\alpha + \beta) = b/a$

16. If $\dfrac{\sin^4 A}{a} + \dfrac{\cos^4 A}{b} = \dfrac{1}{a+b}$, prove that $\dfrac{\sin^8 A}{a^3} + \dfrac{\cos^8 A}{b^3} = \dfrac{1}{(a+b)^3}$

17. If $a\sin^2\theta + b\cos^2\theta = m$, $b\sin^2\phi + a\cos^2\phi = n$ and $a\tan\theta = b\tan\phi$ then show that $\dfrac{1}{m} + \dfrac{1}{n} = \dfrac{1}{a} + \dfrac{1}{b}$

WORKSHEET-I

1. $\left(\frac{1}{3}+3i\right)^3$
2. $\left(-2-\frac{1}{3}i\right)$
3. $\sqrt{5}+3i$
4. Express the following expression in the form of a + ib :
$$\frac{(3+i\sqrt{5})(3-i\sqrt{5})}{(\sqrt{3}+\sqrt{2i})-(\sqrt{3}-i\sqrt{2})}$$
5. Convert the complex number $\frac{-6}{1+i\sqrt{3}}$ into polar form. $[8\left(\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}\right)]$
6. – 1 – i
7. $\sqrt{3}+i$
8. Find the modulus and argument of the complex numbers:
   (i) $\frac{1+i}{1-i}$
   (ii) $\frac{1}{1+i}$ [(i) $\frac{\pi}{2}$ (ii) $\frac{-\pi}{4}$]
9. Find real θ such that $\frac{3+2i\sin\theta}{1-2i\sin\theta}$ is purely real. $[\theta = n\pi, n\in Z]$
10. Convert the complex number $\frac{i-1}{\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}}$ in the polar form. $[\sqrt{2}\left(\cos\frac{5\pi}{12}+i\sin\frac{5\pi}{12}\right)]$
11. For any two complex numbers z_1 and z_2, prove that Re $(z_1 z_2)$ = Re z_1 Re z_2 – Imz_1 Imz_2.
12. If $x-iy=\sqrt{\frac{a-ib}{c-id}}$ prove that $\left(x^2+y^2\right)^2=\frac{a^2+b^2}{c^2+d^2}$
13. Convert the following in the polar form:
    (i) $\frac{1-7i}{(2-i)^2}$
    (ii) $\frac{1+3i}{1-2i}$
14. Let $z_1 = 2 - i$, $z_2 = -2 + i$. Find
    (i) $\text{Re}\left(\frac{z_1 z_2}{z_1}\right)$
    (ii) $\text{Im}\left(\frac{1}{z_1\bar{z}_1}\right)$
15. If $(x + iy)^3 = u + iv$, then show that $\frac{u}{x}+\frac{v}{y}=4(x^2-y^2)$
16. If α and β are different complex numbers with β =1, then find $\left|\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right|$
17. If (a + ib) (c + id) (e + if) (g + ih) = A + iB, then show that
    $(a^2 + b^2)(c^2 + d^2)(e^2 + f^2)(g^2 + h^2) = A^2 + B^2$

WORKSHEET-II

1. Find the value of $\dfrac{i^{592}+i^{590}+i^{588}+i^{586}+i^{584}}{i^{582}+i^{580}+i^{578}+i^{576}+i^{574}}$. [–1]

2. If $x=a+b,\ y=a\alpha+b\beta$ and $z=a\beta+b\alpha,$ where α and β are complex cube roots of unity, show that $xyz=a^3+b^3.$

3. Prove that $\left(\dfrac{-1+i\sqrt{3}}{2}\right)^{17}+\left(\dfrac{-1-i\sqrt{3}}{2}\right)^{17}=-1.$

4. Graph the complex numbers on the Argand plane $3+2i,\ -4-5i,\ -5,\ 1-2i$

5. Express the numbers in the form $r(\cos\theta+i\sin\theta)$:

   (i) $1+i\tan\alpha$ [$\sec\alpha(\cos\alpha+i\sin\alpha)$]

   (ii) $1-\sin\alpha+i\cos\alpha$ [$r(\cos\theta+i\sin\theta),\ f=\sqrt{2}\left(\cos\dfrac{\alpha}{2}-\sin\dfrac{\alpha}{2}\right),\ \theta=\dfrac{\pi}{4}+\dfrac{\alpha}{2}$]

6. Find the square root of following:

   (i) $7-24i$ [$\pm(4-3i)$]

   (ii) $-5+12i$ [$\pm(2+3i)$]

   (iii) $4ab-2(a^2-b^2)i$ [$\pm[(a+b)-i(a-b)]$]

   (iv) $x^2+\dfrac{1}{x^2}+4i\left(x-\dfrac{1}{x}\right)-6$ [$\pm\left(x-\dfrac{1}{x}+2i\right)$]

7. Find x and y in the following equations:

   (i) $(x+iy)(2-3i)=4+i$

   (ii) $\dfrac{(1+i)x-2i}{3+i}+\dfrac{(2-3i)y+i}{3-i}=i$ [(i) $x=\dfrac{5}{13},\ y=\dfrac{14}{13}$ (ii) $x=3,\ y=-1$]

8. Express the following in the form of $a+ib$

   (i) $\dfrac{5+\sqrt{2}i}{1-\sqrt{2}i}$

   (ii) $(1-i)^4$ [(i) $1+2\sqrt{2}i$ (ii) -4]

9. For any two complex numbers z_1 and $z_2,$ prove that $\operatorname{Re}(z_1z_2)=\operatorname{Re}z_1\operatorname{Re}z_2-\operatorname{Im}z_1\operatorname{Im}z_2.$

10. If $(a+ib)(c+id)(e+if)(g+ih)=A+iB,$ then find value of $(a^2+b^2)(c^2+d^2)(e^2+f^2)(g^2+h^2).$ [A^2+B^2]

11. Find the value of $\sqrt[4]{-64}.$ [$\pm2(1\pm i)$]

12. Find the real θ such that $\dfrac{3+2i\sin\theta}{1-2i\sin\theta}$ is purely real. [$\theta=n\pi$]

13. Find the conjugate of $\frac{(3-2i)(2+3i)}{(1+2i)(2-i)}$. [$\frac{63}{25}+\frac{16}{25}i$]

14. Find modulus of the following complex numbers:

(i) $\frac{1+2i}{1-3i}$

(ii) $\frac{1+i}{1-i}-\frac{1-i}{1+i}$ [(i) $\frac{1}{\sqrt{2}}$ (ii) 2]

15. Find argument of following complex numbers:

(i) $\frac{1+i}{1-i}$

(ii) $-\sqrt{3}-i$ [(i) $\frac{\pi}{2}$ (ii) $-\frac{5\pi}{6}$]

16. Put the following complex numbers in polar form :

(i) $\frac{1+2i}{1-3i}$ [$\frac{1}{\sqrt{2}}\left(\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4}\right)$]

(ii) $1+i$ [$\sqrt{2}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)$]

17. If $|2z-1|=|z-2|$, then find value of $|z|$. [1]

18. If $x=\sqrt{2}\,i-1$ find the value of $x^4+4x^3+6x^2+4x+9.$ [12]

19. Express $(1+a^2)(1+b^2)$ as sum of two squares. [$(1-ab)^2+(a+b)^2$]

20. Find the value of $\frac{(1+i)^{4n+5}}{(1-i)^{4n+3}}$. [2]

21. If $z_1=2-i,\quad z_2=1+i,$ find $\left|\frac{z_1+z_2+1}{z_1-z_2+1}\right|$ [$\frac{4\sqrt{5}}{5}$]

22. Let $z_1=2-i,\ z_2=-2,$ find

(i) $\text{Re}\left(\frac{z_1z_2}{\bar{z}_1}\right)$ (ii) $\text{Im}\left(\frac{1}{z_1\bar{z}_1}\right)$

[(i) $\frac{-2}{5}$ (ii)0]

23. If $(x+iy)^3=a+ib,$ then show that $\frac{a}{x}+\frac{b}{y}=4(x^2-y^2)$

24. If α and β are different complex numbers with $|\beta|=1,$ then find $\left|\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right|$. [1]

25. Find the number of non-zero integral solutions of the equation $|1-i|^x=2^x.$ [0]

WORKSHEET-III

1. Express the following in the form $a+ib$:

(i) $(-5i)\left(\frac{1}{8}i\right)$ $[\frac{5}{8}+0i]$

(ii) $(-i)(2i)\left(-\frac{1}{8}i\right)^3$ $[0+\frac{1}{256}i]$

(iii) $(5i)\left(-\frac{3}{5}i\right)$ $[3+0i]$

(iv) i^9+i^{19} $[0+0i]$

(v) i^{-39} $[0+1.i]$

(vi) $(1-i)^4$ $[-4+0i]$

2. Express each of the following in the form $a+ib$

(i) $\left(-2-\frac{1}{3}i\right)^3$ $[-\frac{22}{3}-\frac{107}{27}i]$

(ii) $(5-3i)^3$ $[-10-198i]$

(iii) $(-\sqrt{3}+\sqrt{-2}(2\sqrt{3}-i)$ $[(\sqrt{2}-6)+(\sqrt{3}+2\sqrt{6})i]$

3. Express each one of the following

(i) $\frac{1}{3-4i}$ $[\frac{3}{25}+\frac{4}{25}i]$

(ii) $\frac{5+4i}{4+5i}$ $[\frac{40}{41}-\frac{9}{41}i]$

(iii) $\frac{(1+i)^2}{3-i}$ $[-\frac{1}{5}+\frac{3}{5}i]$

(iv) $\frac{(3-2i)(2+3i)}{(1+2i)(2+i)}$

(v) $\frac{1}{-2+\sqrt{-3}}$ $[-\frac{2}{7}-\frac{\sqrt{3}}{7}i]$

(vi) $\left(\frac{1}{1-2i}+\frac{3}{1+i}\right)\left(\frac{3+4i}{2-4i}\right)$ $[\frac{1}{4}+\frac{9}{4}i]$

(vii) $\frac{1}{1-\cos\theta+2i\sin\theta}$ $[\left(\frac{1-\cos\theta}{2-2\cos\theta+3\sin^2\theta}\right)+i\left(\frac{-2\sin\theta}{2-2\cos\theta+3\sin^2\theta}\right)]$

(viii) $\frac{(3+i\sqrt{5})(3-i\sqrt{5})}{(\sqrt{3}+\sqrt{2}i)-\sqrt{-3}-i\sqrt{2})}$ $[0-\frac{7}{\sqrt{2}}i]$

4. Express $(1-2i)^{-3}$ in the standard form $a+ib$ [$\frac{-11}{125}-\frac{2}{125}i$]

5. Perform the indicated operation and find the result in the form $a+ib$

(i) $\frac{2-\sqrt{-25}}{1-\sqrt{-16}}$ [$\frac{22}{17}+\frac{3}{17}i$]

(ii) $\frac{3-\sqrt{-16}}{1-\sqrt{-9}}$ [$\frac{3}{2}+\frac{1}{2}i$]

6. Find the real values of *x* and *y*, if

(i) $(3x-7)+2iy=-5y+(5+x)i$ [$x=-1,\ y=2$]

(ii) $(1-i)x+(1+i)y=1-3i$ [$x=2, y=-1$]

(iii) $(x+iy)(2-3i)=4+i$ [$x=\frac{5}{13}, y\frac{14}{13}$]

(iv) $\frac{x-1}{3+i}+\frac{y-1}{3-i}=i$ [$x=-4, y=6$]

7. Find real values of x and y for which the following equalities hold:

(i) $(1+i)y^2+(6+i)=(2+i)x$ [$x=5, y=-2$]

(ii) $(x^4+2xi)-(3x^2+iy)=(3-5i)+(1+2iy)$ [$x=-2, y=1/3$]

8. If $(x+iy)^{1/3}=a+ib, x, y, ab \in R$. Show that $\frac{x}{a}+\frac{y}{b}=4(a^2+b^2)$. [$4(a^2-b^2)$]

9. Express the following complex numbers in the standard form. Also, find their conjugate:

(i) $\frac{1-i}{1+i}$ [$\bar{z}=0+i$]

(ii) $\frac{(1+i)^2}{3-i}$ [$\bar{z}=-\frac{1}{5}-\frac{3}{5}i$]

(iii) $\frac{(2+3i)^2}{2-i}$ [$\bar{z}=-\frac{22}{5}-\frac{19}{5}i$]

10. Find real values of *x* and *y* for which the complex numbers $-3+ix^2y$ and x^2+y+4i are conjugate of each other [$x=-1, y=-4$]

11. Find the real numbers *x* and *y* if $(x-iy)(3+5i)$ is the conjugate of $-6-24i$. [$x=3, y=-3$]

12. If $\frac{(a+i)^2}{(2a-i)}=p+iq$, show that: $p^2+q^2=\frac{(a^2+1)^2}{(4a^2+1)}$

13. If $a+ib=\frac{(x+i)^2}{2x^2+1}$, prove that $a^2+b^2=\frac{(x^2+1)^2}{(2x^2+1)^2}$

14. If $x+iy=\sqrt{\frac{a+ib}{c+id}}$, prove that: $(x^2+y^2)^2=\frac{a^2+b^2}{c^2+d^2}$

15. If $(1+i)(1+2i)(1+3i)...(1+ni)=(x+iy)$, show that: 2.5.10…. $(1+n^2)=x^2+y^2$.

16. If z_1, z_2 are complex numbers such that $\frac{2z_1}{3z_2}$ is purely imaginary number, then find $\left|\frac{z_1-z_2}{z_1+z_2}\right|$. [1]

17. If $x=-5+2\sqrt{-4}$, find the value of $x^4+9x^3+35x^2-x+4$ [–160]

18. Find the value of x^3+7x^2-x+16, when $x=1+2i$ [$-17+24i$]

19. Prove that: $x^4+4=(x+1+i)(x+1-i)(x-1+i)(x-1-i)$

20. Find the least positive value of n, if $\left(\frac{1+i}{1-i}\right)^n=i$ [4]

21. Find real θ such that $\frac{3+2i\sin\theta}{1-2i\sin\theta}$ is purely real. [$\theta=n\pi, n\in z$]

22. If $z=2-3i$, show that $z^2-4z+13=0$ and hence find the value of $4z^3-3z^2+169$. [0]

23. If α and β are different complex numbers with $|\beta|=1$, find $\left|\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right|$ [1]

24. Find all non-zero complex numbers z satisfying $\bar{z}=iz^2$. [$z=0, i, \frac{\sqrt{3}}{2}-\frac{1}{2}i, -\frac{\sqrt{3}}{2}-\frac{1}{2}i$]

25. If $iz^3+z^2-z+i=0$, then show that $|z|=1$

26. Solve that equation $z^2+|z|=0$, where z is a complex number

27. Find the multiplicative inverse of the following complex numbers:

(i) $1-i$ [$\frac{1}{2}+\frac{1}{2}i$]

(ii) $(1+i\sqrt{3})^2$ [$-\frac{1}{8}-i\frac{\sqrt{3}}{8}$]

(iii) $4-3i$ [$\frac{4}{25}+\frac{3}{25}i$]

(iv) $\sqrt{5}+3i$ [$\frac{\sqrt{5}}{14}-\frac{3i}{14}$]

28. Evaluate the following:

(i) $x^4-4x^3+4x^2+8x+44$, when $x=3+2i$

(ii) $x^6+x^4+x^2+1$, when $x=\frac{1+i}{\sqrt{2}}$ [(i)12(ii)0]

29. If $z_1=2-i,\ z_2=1+i$, find $\left|\frac{z_1+z_2+1}{z_1-z_2+i}\right|$ [$\frac{4}{\sqrt{2}}$]

30. If $z_1=2-i,\ z_2=1+i$, find

(i) $\text{Re}\left(\frac{z_1 z_2}{\bar{z}_1}\right)$ (ii) $\text{Im}\left(\frac{1}{z_1\bar{z}_1}\right)$ [(i)$-\frac{2}{5}$ (ii)0]

31. Find the real values of q for which the complex number $\frac{1+i\cos\theta}{1-2i\cos\theta}$ is purely real.

[$\theta=2n\pi\pm\frac{\pi}{2}, n\in Z$]

32. If $\left(\frac{1+i}{1-i}\right)^3-\left(\frac{1-i}{1+i}\right)^3 = x+iy$, find (x, y) [(0, –2)]

33. If $\frac{(1+i)^2}{2-i}=x+iy$, find $x+y$. [$\frac{2}{5}$]

34. If $\left(\frac{1-i}{1+i}\right)^{100} = a+ib$, find (*a*, *b*). [(1, 0)]

35. If $a=\cos\theta+i\sin\theta$, find the value of $\frac{1+a}{1-a}$ [$i\cot\frac{\theta}{2}$]

36. Find the square roots of the following
(i) $7-24i$ (ii) $5+12i$
[(i) $\pm(4-3i)$ (ii) $\pm(3+2i)$]

37. Find the square root of *i*.
[$\pm\frac{1}{\sqrt{2}}(1+i)$]

38. Write the following complex numbers in the polar form:

(i) $-3\sqrt{2}+3\sqrt{2}i$ [$6\left(\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4}\right)$]

(ii) $1+i$ [$\sqrt{2}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)$]

(iii) $-1-i$ [$\sqrt{2}\left(\cos\frac{3\pi}{4}-i\sin\frac{3\pi}{4}\right)$]

(iv) $1-i$ [$\sqrt{2}\left(\cos\frac{\pi}{4}-i\sin\frac{\pi}{4}\right)$]

39. Put the complex number $\frac{1+7i}{(2-i)^2}$ in the form $r(\cos\theta+i\sin\theta)$, where r is a positive real number and $-\pi<\theta\le\pi$. [$\sqrt{2}\left(\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4}\right)$]

40. Find the modulus and argument of the following complex numbers and convert them in polar form:

(i) $\frac{1+2i}{1-3i}$ [$\frac{1}{\sqrt{2}}\left(\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4}\right)$]

(ii) $\frac{i-1}{\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}}$ [$\sqrt{2}\left(\cos\frac{5\pi}{12}+i\sin\frac{5\pi}{12}\right)$]

(iii) $\frac{1+3i}{1-2i}$ [$\sqrt{2}\left(\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4}\right)$]

41. Find the modulus and argument of the following complex numbers and hence express each of them in the polar forms

(i) $1+i$ [$\sqrt{2}(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4})$]

(ii) $\sqrt{3}+i$ [$2(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6})$]

(iii) $1-i$ [$\sqrt{2}(\cos\frac{\pi}{4}-i\sin\frac{\pi}{4})$]

(iv) $\frac{1-i}{1+i}$ [$(\cos\frac{\pi}{2}-i\sin\frac{\pi}{2})$]

(v) $\dfrac{1}{1+i}$ [$\dfrac{1}{\sqrt{2}}(\cos\dfrac{\pi}{4} - i\sin\dfrac{\pi}{4})$]

(vi) $\dfrac{1+2i}{1-3i}$ [$\dfrac{1}{\sqrt{2}}(\cos\dfrac{3\pi}{4} - i\sin\dfrac{3\pi}{4})$]

(vii) $\sin 120^\circ - i\cos 120^\circ$ [$\cos\dfrac{\pi}{6} + i\sin\dfrac{\pi}{6}$]

(viii) $\dfrac{-16}{1+i\sqrt{3}}$ [$8\left(\cos\dfrac{2\pi}{3} + i\sin\dfrac{2\pi}{3}\right)$]

[(

42. Express $\sin\dfrac{\pi}{5} + i\left(1-\cos\dfrac{\pi}{5}\right)$ in polar form [$2\sin\dfrac{\pi}{10}\left(\cos\dfrac{\pi}{10} + i\sin\dfrac{\pi}{10}\right)$]

43. Write $(i^{25})^3$ in polar form [$\cos\dfrac{\pi}{2} - i\sin\dfrac{\pi}{12}$]

44. If $\pi < \theta < 2\pi$ and $z = 1+\cos\theta + i\sin\theta$, then write the value fo $|z|$ [$-2\cos\dfrac{\theta}{2}$]

45. If n is any positive integer, write the value of $\dfrac{i^{4n+1} - i^{4n-1}}{2}$. [$i$]

46. Write the argument of $-i$. [$\dfrac{3\pi}{2}$ or $-\dfrac{\pi}{2}$]

WORKSHEET-I

1. Solve the following systems of inequations graphically:

   (i) $2x+y\geq 8,\ x+2y\geq 8,\ x+y\leq 6$

   (ii) $12+12y\leq 840,\ 3x+6y\leq 300,\ 8x+4y\leq 480,\ x\geq 0,\ y\geq 0$

   (iii) $x+2y\leq 40,\ 3x+y\geq 30,\ 4x+3y\geq 60,\ x\geq 0,\ y\geq 0$

   (iv) $5x+y\geq 10,\ 2x+2y\geq 10,\ x+4y\geq 12,\ x\geq 0,\ y\geq 0$

2. Show that the following system of linear equations has no solutions:

   $x+2y\leq 3$, $3x+4y\geq 12$, $x\geq 0$, $y\geq 1$

3. $3x-2y\leq x+y-8$

3. Find the pairs of consecutive odd positive integers, both of which are smaller than 10, such that their sum is more than 11.

4. To receive grade ‘A’ in a cource, one must obtain an average of 9 marks or more in five papers each of 100 marks. If Shikha scored 87, 95, 92 and 94 marks in first four papers, find the minimum marks that she must score in the last paper to get grade ‘A’ in the course.

5. How many litres of water will have to be added to 1125 litres of the 45% solution of acid so that the resulting mixture will contain more than 25% but less than 3% acid content?

6. A solution of 8% boric acid is to be diluted by adding a 2% boric acid solution to it. The resulting mixture is to be more than 4% but less than 6% boric acid. If there are 64 litres of the 8% solution, how many litres of 2% solution will have to be added?

7. The water acidity in a pool is considered normal when the average pH reading of three daily measurements is between 7.2 and 7.8. If the first two pH reading are 7.48 and 7.85, find the range of pH value for the third reading that will result in the acidity level being normal.

8. $|x+1|+|x|>3$

9. $|x+1|+|x-2|+|x-3|\geq 6$

10. $\dfrac{1}{|x|-3}\leq\dfrac{1}{2}$

11. $1\leq|x-2|\leq 3$